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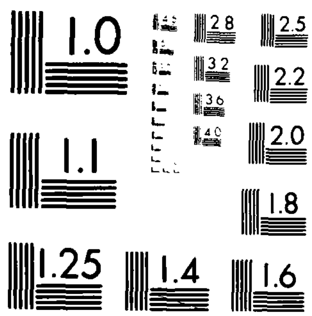

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ACQUISITION OF PROBLEM-SOLVING SKILLS IN BASIC ELECTRICITY AND ELECTRONICS

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The objectives of this effort were to (1) identify the knowledge components that students in the Navy's BE/E schools need to solve problems involving DC circuits, (2) identify which of those components students fail to acquire in BE/E, and (3) explore ways to instruct students in these missing components. First, a theoretical task analysis was performed to identify the knowledge required to perform successfully on the first six BE/E modules. Next, two experiments were conducted. The purpose of Experiment I was to test model predictions. Verbal protocols were obtained from seven subjects as they

learned to solve DC circuit problems from the BE/E course and subjects' patterns of performance were compared to the predicted patterns of performance. The purpose of Experiment II was to determine whether the errors students made in Experiment I could be prevented by providing them with a model of a concrete analogy of circuit relations in which the constraints are salient and therefore more likely to be understood and used in solving problems.

Experiment I showed that providing students with the minimum knowledge and procedures to solve a selected set of problems is not necessarily sufficient. Unless students understand the constraints underlying those procedures, they fail to integrate these constraints in their problem-solving procedures. The results of Experiment II suggest that the analogy facilitated performance on series and parallel problems by making circuit constraints more salient and by providing subjects with simple procedures that take those constraints explicitly into account.

FOREWORD

This research and development was principally funded under exploratory development task area RF63.522.001-011 (Assessment and Enhancement of Prerequisite Skills), work unit 03.02 (Enhancement of Computational Capabilities). Secondary funding was provided by the Office of Naval Research under research project R-R042-06-02. The objectives of work unit 03.02 are to identify mathematical skill deficiencies among Navy electronics personnel, to determine the causes of such deficiencies, and to develop instructional strategies to improve the efficiency and job relevance of Navy electronics training.

The objective of this study was to explore the acquisition of problem-solving skills in basic electricity and electronics. The results are intended for use by those involved in the development of instructional delivery techniques.

The contracting officer's technical representative was Dr. Meryl S. Baker.

J. W. RENARD
Captain, U.S. Navy
Commanding Officer

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Technical Director

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SUMMARY

Problem

The Navy's Basic Electricity and Electronics (BE/E) Preparatory Schools are responsible for training personnel in the knowledge and applied skills of basic electricity and electronics that are entry-level prerequisites for a number of Navy ratings. Unless students demonstrate competency in these prerequisites, they cannot continue with more advanced training. Therefore, the academic attrition rate for BE/E schools--sometimes as high as 30 percent--is of great concern. Setback rates are also high. BE/E trainees can begin experiencing difficulties during the first six BE/E modules, which concern direct current (DC) circuit problems.

Objective

The objectives of this effort were to (1) identify the components of knowledge required to solve problems involving DC circuits, (2) identify which of those components students fail to acquire in BE/E, and (3) explore ways to instruct students in these missing components.

Approach

A theoretical task analysis was performed to identify the knowledge required to perform successfully on the first six BE/E modules. The analysis was in the form of a production system model that solved the circuit problems in the modules.

Two experiments were conducted. The purpose of Experiment I was to test model predictions. Verbal protocols were obtained from seven subjects as they learned to solve DC circuit problems from the BE/E course and subjects' patterns of performance were compared to the predicted patterns of performance. The purpose of Experiment II was to determine whether the errors students made in Experiment I could be prevented by providing them with a model of a concrete analogy of circuit relations in which the constraints are salient and therefore more likely to be understood and used in solving problems. Materials and procedures were similar to those used in Experiment I.

Results

Results of Experiment I were generally consistent with model predictions. The minimum components of knowledge required to solve DC circuit problems are explicitly taught in BE/E; however, subjects failed to understand circuit constraints underlying the instructed procedures. As a result, they (1) failed to make critical distinctions between the elements in circuit equations, (2) overgeneralized procedures in ways that violate circuit constraints, and (3) overextended analogical models of electrical circuits taught in BE/E. Most of these deficiencies are not detected by the majority of problems in the BE/E programmed instruction and progress tests.

Results of Experiment II indicate that the analogy did facilitate performance on series and parallel problems. However, errors persisted on problems that required solving for the voltage drop across a series resistor.

Conclusions

Experiment I showed that providing students with the minimum knowledge and procedures to solve a selected set of problems is not necessarily sufficient. Unless students understand the constraints underlying those procedures, they fail to integrate these constraints in their problem-solving procedures. Even when students do not understand DC circuit relations, they can pass the existing progress tests in the course.

The results of Experiment II suggest that the chips analogy facilitated performance on series and parallel problems by making circuit constraints more salient and by providing subjects with simple procedures that take those constraints explicitly into account. Further, it appears that showing subjects how this analogy corresponds to circuit equations and qualitative rules may have enabled them in some cases to incorporate these constraints into procedures for solving problems correctly without explicitly using the analogy. However, results also suggest that further research is needed to identify ways of connecting students' understanding of the constraints on procedures for manipulating concrete materials to their understanding of analogous procedures for manipulating symbols.

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INTRODUCTION

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The Navy's Basic Electricity and Electronics (BE/E) Preparatory Schools are responsible for training personnel in the knowledge and applied skills of basic electricity and electronics that are entry-level prerequisites for a number of Navy ratings. Unless students demonstrate competency in these prerequisites, they cannot continue with more advanced training. Therefore, the academic attrition rate for BE/E Schools--sometimes as high as 30 percent (Atwater & Abrahams, 1980)--is of great concern. Setback rates are also high.

Many BE/E students experience considerable difficulty completing the first six instructional modules, which cover problems involving direct current (DC) circuits. Yet the minimum components of knowledge required to solve these problems are few and are taught explicitly in the BE/E course.

Objective

The objectives of this effort were to (1) identify the components of knowledge required to solve problems involving DC circuits, (2) identify which of those components students fail to acquire in BE/E, and (3) explore ways to instruct students in these missing components.

APPROACH

A theoretical analysis of the knowledge required to solve DC circuit problems was conducted. The analysis was in the form of a production system model that solves the circuit problems in the BE/E course. It was proposed that the knowledge and procedures included in the model represent the minimum knowledge required to solve all problems correctly. The model was then compared to the content of the BE/E course and modified by either (1) deleting a specific component of knowledge because it is not taught explicitly in the course but only implicitly constrains the examples, or (2) adding one or more components that may result as generalizations from the problems set in the course. These alternative versions were then used to predict specific errors, if any, that would occur.

Two experiments were conducted. The purpose of Experiment I was to test model predictions. Protocols were obtained from seven subjects as they learned to solve DC circuit problems from the Navy BE/E course and subjects' patterns of performance were compared to the predicted patterns of performance. The purpose of Experiment II was to determine whether the errors students made in Experiment I could be prevented by providing them with a model of a concrete analogy of circuit relations in which the constraints are salient and therefore more likely to be understood and used in solving problems.

THEORETICAL ANALYSIS

Model Performance

The theoretical analysis was in the form of a production system model that solves problems about DC series, parallel, and combination circuits. Figure 1 is an example of a problem involving a series circuit that has two resistors, R_1 and R_2. The values of the total voltage (E_t) applied by the battery (or "source"), the total current (I_t), and the resistance of the second resistor (R_2) are given; the task is to solve for the voltage drop across R_1 (E_{R_1}). The letters "A" through "F" indicate the endpoints of the line segments in the schematic.

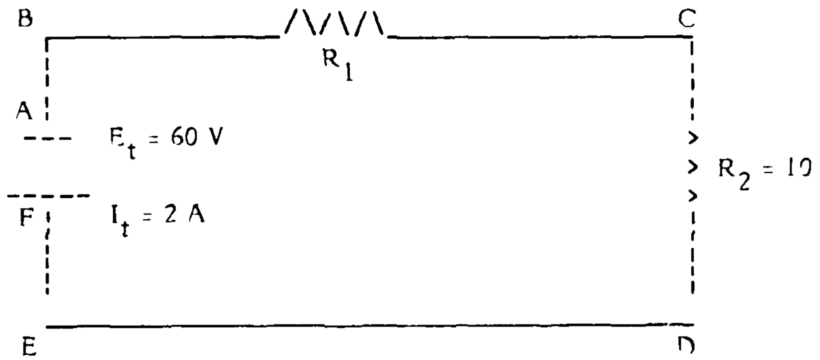

Figure 1. Determining the voltage drop across R_1 (adapted from Problem 18, BE/E Module 2-7).

Problems that require an actual quantity as a response, as in Figure 1, are referred to as quantitative problems. The model can also solve qualitative problems, those that require reasoning about the effects of qualitative changes in circuit values. For example, "If the total voltage increases, what is the effect on the total current?" (Does it increase, decrease, or remain the same?)

The general framework for the model's solution process is shown in Figure 2. When the model receives a problem, it first applies its parsing procedures to build a representation of the series and/or parallel relations between the resistors. All circuit values given in the problem are included in the representation. Next the model selects the appropriate strategy for the question being asked--quantitative problems are solved using a strategy known as "backward search," and qualitative problems are solved using a

strategy of "constraint propagation." The model then applies the strategy to solve the problem, using the information in its initial representation, its knowledge of the circuit equations that specify the relations between current, voltage, and resistance in DC circuits, and a WHOLE-PARTS schema for identifying the relations between quantities in equations.

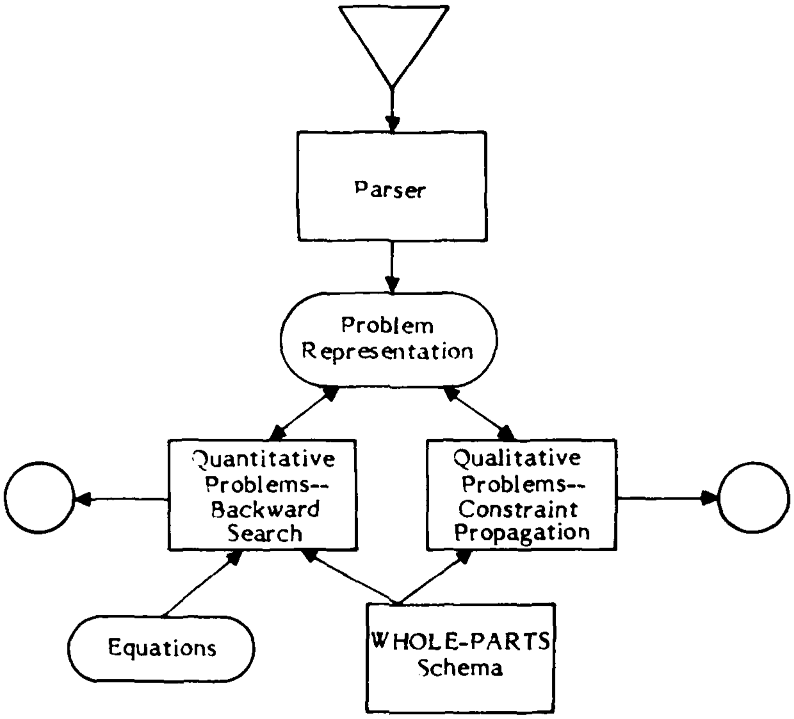

Figure 2. General framework for the model's solution process.

For example, to solve for the voltage drop across R_1 in Figure 1, the problem statement presented to the model would include the following information:

1. Line segments AB, BC, CD, DE, and EF are connected by 90 degree angles.

2. Line segments AB and EF respectively are joined to the negative and positive terminals of the voltage source.

3. Line segments BC and CD each contain one resistor.

4. The values of the total voltage (E_t) applied by the source, the total current (I_t), and the resistance at the second resistor (R_2) are 60V, 2A and 10 respectively.

5. The question to be answered.

The first thing the model does when it receives the problem is to parse the information about the circuit schematic into a representation of the relations between the source and resistors R_1 and R_2. The parsing procedure begins with the line segment that has an endpoint joined to the negative terminal--in this case, AB--and continues around the circuit, using information about shared endpoints to identify successive line segments. As long as each line segment shares an endpoint with only one other line segment, the model assumes that any resistor(s) contained within those line segments are in series with the source. (The implications of this heuristic will become clearer in the discussion of parsing parallel circuits.) The model therefore represents R_1 and R_2 as being in series with the source, as shown in the semantic network representation in Figure 3.

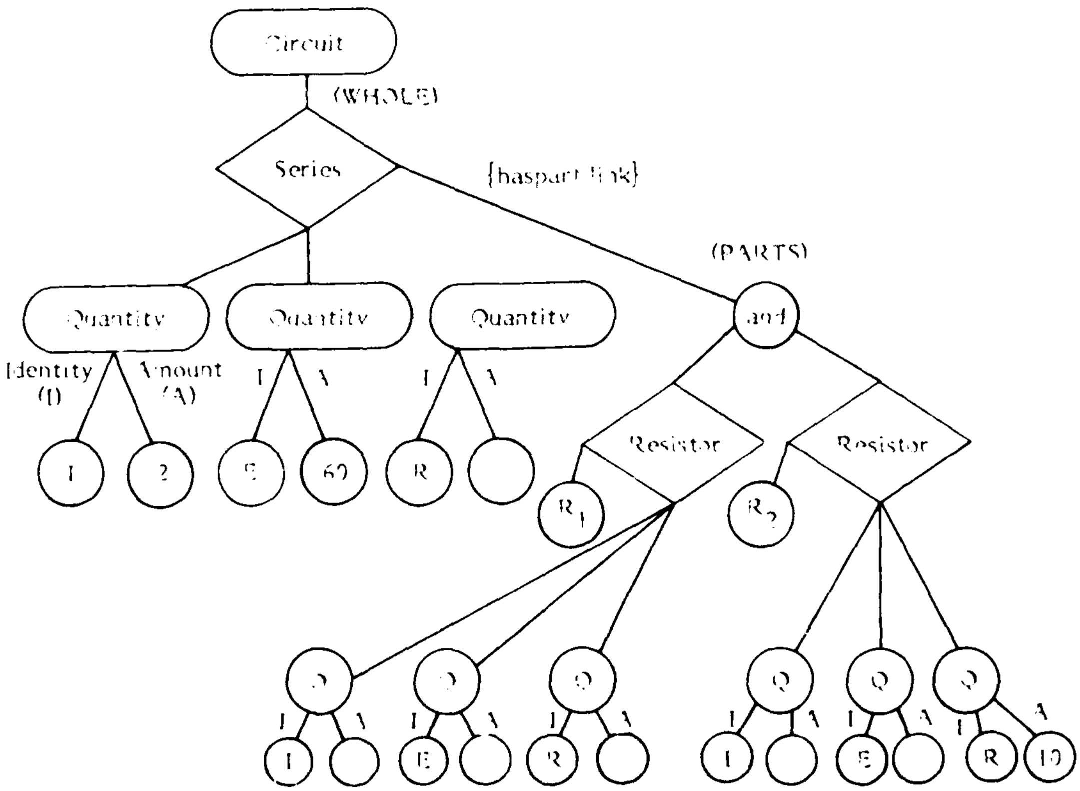

Figure 3. Semantic network representation.

The form of this representation is similar to other node-link structures used in recent theories of language understanding (e.g., Anderson, 1976; Norman & Rumelhart, 1975; Schank & Abelson, 1977). It consists of three main clusters of nodes and links organized hierarchically to represent different levels of circuit relations. A cluster is defined as a series, parallel, or resistor node and its corresponding quantity nodes. The cluster represented by the series node is at the highest level. The link to circuit indicates that this cluster represents the values of the total circuit. The two resistor nodes distinguished by identity links to R_1 and R_2 are at the next lower level. The "haspart" link between these nodes and the series node indicates that series is a whole that contains R_1 and R_2 as parts.

The resistance values for each cluster are represented as quantities having an identity (I, E, or R) and an amount. The values of the quantities given in the problem are included in the representation at this time: I_t has a value of 2 A, E_t has a value of 60V, and the resistance at R_2 is 10. Additional values are included as they become known during problem solving.

Once the model has constructed the representation in Figure 3, it determines the appropriate general strategy for solving the problem based on the type of question asked. If the question requires a quantitative response like that in Figure 1, the model selects a strategy known as "backward search" (Newell & Simon, 1972). If the problem involves a change in one of the circuit values and the question is to predict the qualitative effect of this change on other circuit values, the model selects a strategy known as "constraint propagation." The model's performance on quantitative and qualitative problems is described below.

Quantitative Problems

The model's backward search strategy is similar to the novice problem-solving strategies identified in several domains (cf., Chi, Glaser, & Rees, 1981). In the model, the basic strategy involves (1) identifying the quantity in the question as the goal of the problem, (2) selecting an equation that contains this goal, and (3) substituting the quantities given in the problem statement for the corresponding required variables in the equation. If one of the required variables is still unknown, then that quantity becomes the immediate goal and the backward search strategy is applied recursively to determine its value. Once the values of all the required variables in an equation are known, the model applies its WHOLE-PARTS schema to solve for the goal quantity.

During problem solving, the model keeps track of its goals by means of a goal stack. Initially, there is only one goal on the stack--to solve for the quantity in the question. If one of the required variables is identified as a goal, it is added to the goal stack on top of any previous goals. Once the top goal on the stack--the immediate goal--is satisfied, it is removed from the stack and the goal immediately underneath it becomes the new immediate goal. Thus, goals are satisfied on a "last in, first out" basis. Problem solving continues until there are no more goals in the stack (i.e., until the quantity in the question is known).

Figure 4 provides a more formal description of the model's backward search strategy. The strategy consists of a set of productions, of IF-THEN rules: IF a particular condition is true, THEN perform the corresponding action. Problem solving is accomplished by means of a series of iterations or "cycles" through this set of productions. On each cycle,

the conditions of the productions are tested in order, beginning with P_1, until one of the conditions applies to the immediate problem situation. The action of that production is then performed, or "fired," and the cycle is complete. Performing an action usually changes the current problem situation, causing a different production to match on the next cycle.

```
P1.   IF      (INPUT = QQ?)
      THEN    (SETGOAL QQ)
P2.   IF      (QQ hasprop Known)
      THEN    (SAY QQ)
              (POPSTACK)
              (STOP)
P3.   IF      (EQ haspart GOAL)
              (EQ hasprop Unused-Cluster)
      THEN    (SELECT EQ)
              (EQ haspart Used-Cluster)
P4.   IF      (EQ hasprop Used-Cluster)
              (SUB-EQ hasprop False)
      THEN    (SUBSTITUTE KQ VAR-EQ)
              (SUB-EQ hasprop True)
P5.   IF      (N (VAR-EQx hasprop Unknown)
              < N (VAR-EQy hasprop Unknown))
      THEN    (PREFER  EQx)
P6.   IF      (N (VAR-EQx hasprop Unknown)
              = N (VAR-EQy hasprop Unknown))
      THEN    (PREFER  EQ1)
P7.   IF      (VAR-EQ hasprop Unknown)
      THEN    (SETGOAL VAR-EQ)
P8.   IF      (Absence (VAR-EQ hasprop Unknown))
      THEN    (SOLVE)
              (GOAL hasprop Known)
P9.   IF      (GOAL hasprop Known)
      THEN    (POPSTACK)
              (SUBSTITUTE)
```

Figure 4. The model's "backward search" strategy.

The first production (P1) in Figure 4 says, "IF the value of QQ (the quantity in the question) is unknown, THEN use the function SETGOAL to put QQ on top of the goalstack, making it the immediate goal." The second production (P2) says, "IF the value of QQ is already known, THEN SAY the value, use a function called POPSTACK to remove the goal from the stack, and STOP."

The next four productions are concerned with selecting equations. P3 says, "IF there is an equation (EQ) that contains the immediate goal, and that equation has not yet been selected within the same cluster as the immediate goal, THEN select that equation." The

cluster restriction precludes the endless loop of selecting and reselecting the same equation.

Once an equation has been selected, P4 attempts to substitute the known quantities (KQ) for the required variables (VAR) in the equation. The variable SUB-EQ (substituted values into equation) is then set equal to True so that P4 will not match again for the same equation. In cases where more than one equation was selected to solve for the GOAL, P5 prefers the equation that has the least number of unknown required variables (VARs). If both equations have equal numbers of unknown VARs, 6 prefers Ohm's law.

After a single equation has been selected and the known quantities have been substituted for the required variables in the equation, the model determines whether it has enough information to solve for the GOAL. P7 says that, "IF anyone of the VARs is unknown, THEN make that variable the immediate goal." P8 says that, "IF all the VARs are known, then call the SOLVE procedure" (discussed below). Once the immediate goal has been satisfied, the last production, P9, "pops" this goal off the stack and, when relevant, substitutes its value into the equation containing the new immediate goal.

The manner in which the model's parsing procedures, problem representation, backward search strategy, and equation interact to solve quantitative circuit problems like that in Figure 1 is illustrated in Table 1 and described as follows. On the first cycle, since the value of the voltage drop across R_1 (E_{R_1}) is unknown, the condition of P1 (Figure 4) matches and the action is to make E_{R_1} the immediate goal. On cycles 2 and 3, the model refers to its knowledge of circuit equations to select an equation (listed in Table 2) that contains this goal. The model's equations are shown in Figure 6. (The model checks only the first five equations in Table 2, since the latter three apply to parallel circuits--not series circuits.) The condition of P3 matches first to EQ1 ($I_{R_1}=E_{R_1}/R_1$) and then to EQ4 ($E_t=E_{R_1}+E_{R_2}$) because both equations contain the immediate goal. On cycles 4 and 5, the condition of P4 is relevant and the model attempts to substitute the known quantities in its problem representation (Figure 3) for the required variables in the equations. No values are substituted for the EQ1 on cycle 4, since the values of I_{R_1} and R_1 are not given in the problem. On cycle 5, the 60 volts are substituted for the value of the total voltage in EQ2, making it the equation with the least number of unknown variables. P5 therefore prefers the second equation over the first on cycle 5.

On cycle 6, the model cannot solve for E_{R_1} since the value of one of the required variables, E_{R_2}, is still unknown. E_{R_2} is identified as the new goal quantity (cycle 7, P7), and the model begins to recursively apply its backward search strategy. On cycle 8, both EQ1 ($I_{R_2}=E_{R_2}/R_2$) and EQ4 ($E_t=E_{R_1}+E_{R_2}$) again contain the goal (E_{R_2}). However, since EQ4 was just applied within the same cluster, only EQ1 satisfies the condition of P3.

Table 1

The Model's Solution to the Problem in Figure 1

Cycle	Production	Action
1	P1	SETGOAL: E_{R_1}
2	P3	SELECT: $I_{R_1}=E_{R_1}/R_1$
3	P3	SELECT: $E_t=E_{R_1}+E_{R_2}$
4	P4	SUBSTITUTE: $I_{R_1}=E_{R_1}/R_1$
5	P4	SUBSTITUTE: $60V=E_{R_1}+E_{R_2}$
6	P5	PREFER: $60V=E_{R_1}+E_{R_2}$
7	P7	SETGOAL: E_{R_2}
8	P3	SELECT: $I_{R_2}=E_{R_2}/R_2$
9	P4	SUBSTITUTE: $I_{R_2}=E_{R_2}/10$
10	P7	SETGOAL: I_{R_2}
11	P3	SELECT: $I_t=I_{R_1}=I_{R_2}$
12	P4	SUBSTITUTE: $2A=I_{R_1}=I_{R_2}$
13	P8	SOLVE: $2A=I_{R_1}=I_{R_2}, I_{R_2}=2A$
14	P9	POPSTACK: E_{R_2} SUBSTITUTE: $2A=E_{R_2}/10$
15	P8	SOLVE: $2A=E_{R_2}/10, E_{R_2}=20V$
16	P9	POPSTACK: E_{R_1} SUBSTITUTE: $60V=E_{R_1}+20V$
17	P8	SOLVE: $60V=E_{R_1}+20V\ E_{R_1}=40V$
18	P2	SAY: 40V POPSTACK STOP

Table 2

The Model's Equations (EQs) for Solving Circuit Problems

Number	Equation	Statement of Relations
All Circuits:		
EQ1	$I_{R_n} = E_{R_n}/R_n$	Current (I) is always equal to the ratio of the voltage (E) over the resistance (R).
EQ2	$P_{R_n} = (I_{R_n})(E_{R_n})$	Power (P) is always equal to the product of the current (I) and the voltage (E).
Series Circuits:		
EQ3	$I_t = I_{R_1} = I_{R_2} = \ldots I_{R_n}$	Total current (I_t) is the same everywhere in a series circuit.
EQ4	$E_t = E_{R_1} + E_{R_2} + \ldots E_{R_n}$	Total voltage (E_t) is equal to the sum of the voltage drops (E_R) across the individual resistors.
EQ5	$R_t = R_1 + R_2 + \ldots R_n$	Total resistance (R_t) is equal to the sum of the resistance values of all the resistors.
Parallel Circuits:		
EQ6	$I_t = I_{R_1} + I_{R_2} + \ldots I_{R_3}$	Total current (I_t) is equal to the sum of the branch currents.
EQ7	$E_t = E_{R_1} = E_{R_2} + \ldots E_{R_3}$	Total voltage (E_t) is equal to the voltage drop across a parallel branch.
EQ8	$R_t = 1/((1/R_1) + (1/R_2) + \ldots (1/R_3))$	Total resistance (R_t) is determined by the reciprocal method.

Note: The first two equations specify the relations that exist among the current, voltage, and resistance values within any cluster in any circuit; the next three, the between-cluster relations in a series circuits; and the last three, the between-cluster relations in a parallel circuit.

On cycle 9, the model retrieves the resistance value of R_2 from its problem representation and substitutes it for the value of R_2 in the equation. However, the model still cannot solve for E_{R_2} because the value of I_{R_2} is still unknown. Therefore, on cycle 10, the condition of P7 matches and the goal is set to determine the value of I_{R_2}. At this point, there are three goals on the goal stack: E_{R_1} is on the bottom, E_{R_2} is next, and I_{R_2}, the immediate goal, is on top.

Next the model selects EQ3 ($I_t = I_{R_1} = I_{R_2}$) to determine I_{R_2} (cycle 11), substitutes the value of the total current into the equation cycle (cycle 12), and solves for the value of I_{R_2} (cycle 13).

The value of I_{R_2} is now part of the model's problem representation (Figure 3). Therefore, on cycle 14, POPSTACK removes I_{R_2} from the goal stack, making E_{R_2} the top goal on the stack and the new immediate goal. The value of I_{R_2} is substituted into the equation that was selected to solve for E_{R_2}.

All the required variables in the equation containing E_{R_2} are now known. The model solves for the value of E_{R_2} (cycle 15, P8) and removes E_{R_2} from the goalstack (cycle 16, P9). E_{R_1} is again the immediate goal and the value of E_{R_2} is substituted into the original equation. Finally, the model solves for the value of E_{R_1} (cycle 17, P8) and reports this value, removes E_{R_1} from the goalstack, and stops (cycle 18, P2).

Next, the manner in which the model's WHOLE-PARTS schema is invoked by the SOLVE procedure to select the appropriate operation to solve the equation is described. The model's WHOLE-PARTS schema shown in Figure 5. The term "schema," as used here, simply refers to a general set of relations and associated procedures that can be used to interpret many specific situations. In the model, the schema is used to interpret the relations between quantities in equations.

The structure of this schema is consistent with recent proposals in problem solving and artificial intelligence (e.g., Anderson, Greeno, Kline, & Neves, 1981; Bobrow & Winograd, 1977). The slots indicate the kinds of objects the schema is designed to interpret; in this case, the objects are a WHOLE and two or more PARTs.

```
Slots:    WHOLE, PART1, PART2, ...PARTn

Schematizing Productions:

    1.  IF      (EQ: X1=X2+X3...+Xn)
        THEN    (X1 isa WHOLE) (X2, X3, Xn isa PARTn)
                (COMBINE: ADD) (SEPARATE: SUBTRACT)

    2.  IF      (EQ: X1=X2/X3)
        THEN    (X2 isa WHOLE) (X1, X2 isa PARTn)
                (COMBINE: MULTIPLY) (SEPARATE: DIVIDE)

Procedural Attachments:

    A1. FIND-WHOLE
        Prerequisite:
          (PART1, PART2, ...PARTn hasprop Known)
        Consequence:
          (WHOLE hasprop Known)
        Performance:
          (COMBINE PART1, PART2, ...PARTn)

    A2. FIND-PART
        Prerequisite:
          (WHOLE, PART2, ...PARTn hasprop Known)
        Consequence:
          (PART1 hasprop Known)
        Performance:
          (SEPARATE WHOLE, PART2, ...PARTn)
```

Figure 5. The model's WHOLE-PARTS schema.

The "schematizing productions" (Anderson, et al., 1981) interpret specific situations in terms of the objects in the schema. Those shown in Figure 5 are relevant to identifying the WHOLEs and PARTs in equations. The first production says that, "IF the equation has the form, X1=X2+X3...+Xn, THEN identify X1 as the WHOLE and the other variables as PARTs, and associate the actions COMBINE and SEPARATE with ADD and SUBTRACT respectively." The second production says that, "IF the equation has the form, X1=X2/X3, THEN interpret the X2 as the WHOLE and X1 and X3 as PARTS, and associate COMBINE with MULTIPLY and SEPARATE with DIVIDE." Other schematizing productions would be used to interpret other situations. However, once the situation has been interpreted into WHOLEs and PARTs, the same FIND-WHOLE and FIND-PART procedural attachments apply to all situations.

Procedural attachments describe procedures that can be performed on the objects in the schema. They are organized in a way proposed by Sacerdoti (1977) with prerequisites, consequences, and actions that are performed to execute the procedure. For the FIND-WHOLE attachment (A1), the prerequisite is that values of the PARTs are known, the consequence is that the value of the WHOLE will be found, and the action is to combine the PARTs. For the FIND-PART attachment (A2), the prerequisite is that the value of the WHOLE and all but one of the PARTs are known, the consequence is that the unknown PART is now known, and the action is to separate the known PART(s) from the WHOLE.

In the model, the WHOLE-PARTs schema is invoked by the SOLVE procedure to interpret the relations between quantities in equations. In the problem solution in Table 1, the SOLVE procedure invoked the WHOLE-PARTs schema to identify the appropriate operation to solve for E_{R_2} in the equation, $A2=E_{R_2}/10$ (cycle 15). The form of this equation matches the condition of the second schematizing production and the WHOLE-PARTs schema is instantiated, with E_{R_2} identified as the WHOLE and 2A and 10 ohms identified as PARTs. Since the unknown is the WHOLE and the FIND-WHOLE procedure identifies, the relevant action is to combine--in this case, multiply--the two PARTs. For the equation, $60V=E_{R_1}+20V$ (cycle 16, Table 1), the unknown E_{R_1} is a PART; therefore, the relevant action is to separate the other PART (20V) from the WHOLE (60V). The actual computation is carried out by the SOLVE procedure.

The knowledge included in the model is sufficient to solve the remaining quantitative problems in the BE/E course.

Qualitative Problems

Unlike quantitative problems, qualitative problems require no numerical response. Voltage or resistance is varied and the task is to predict the effects on other circuit values. For example, referring to the series circuit in Figure 6, what effect would increasing the resistance at R_2 have on the voltage drops across R_1, R_2, and R_3? Would they increase, decrease, or remain the same?

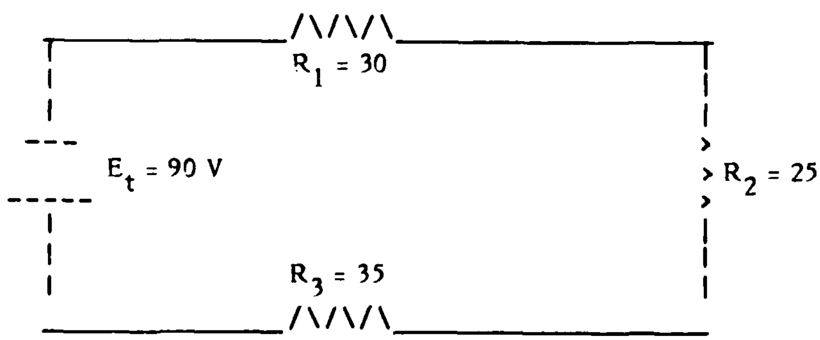

Figure 6. Determining the effects of an increase in R_2 (27.5) (adapted from Problem 11, BE/E Module 4-1).

As indicated previously and shown in Figure 2, the model solves qualitative problems using a process of "constraint propagation." The "constraints" correspond to the qualitative relations embodied in circuit equations and described below. The "propagation" involves applying these constraints to the known changes to determine

additional changes in circuit values. These changes are then used to determine additional relations, etc., until the required relations have been constrained. For example, the effects of increasing R_2 in Figure 6 can be determined by reasoning as follows:

1. If R_2 increases, total resistance increases.

2. If total resistance increases, current decreases.

3. If the current through R_1 and R_3 decreases and their resistance values remain the same, then the voltage drops across R_1 and R_3 decrease.

4. If the voltage drops across R_1 and R_3 decrease and the total voltage remains the same, then the voltage drop across R_2 must have increased.

The remainder of this section describes two versions of a model that reasons qualitatively about the effects of changing circuit values without comparing quantities. In the first version, the model reasons explicitly in terms of circuit equations and WHOLE-PARTS relations to predict the effects of changing a circuit value. In the second version, the equations and WHOLE-PARTS relations only implicitly constrain a set of productions that directly relate changes and effects. Only the second version has been implemented as a running computer model. The first version was simulated by hand.

The qualitative relations that exist between the elements in circuit equations are described by the additional procedural attachments to the model's WHOLE-PARTS schema shown in Figure 7. Procedural attachments A3 and A4 concern situations in which one or more PARTS has changed. For A3, the prerequisite is that one PART changes and the other PARTS remain the same, the consequence is that the WHOLE changes in the same direction, and the action would be to ADJUST the actual value of the WHOLE but this is not necessary for qualitative reasoning. For A4, the prerequisite is that all but one of the PARTs change and the WHOLE remains the same; the consequence is that the one PART must change in the opposite direction of the other PARTS.

Procedural attachments A5 and A6 concern the effects of changing the WHOLE. The prerequisite of A5 is that the WHOLE changes and all but one of its PARTS remain the same; the consequence is that the value of the one PART changes in the same direction as the WHOLE. The prerequisite of A6 is that the WHOLE changes and none of the PARTs remain the same; the consequence is that all the PARTs change in the same direction as the WHOLE.

Additional procedural attachments could be generated to describe other qualitative relations that could exist between a WHOLE and its PARTs, but the four attachments in Figure 7 are sufficient to describe the qualitative effects of changing circuit values. Consider how these attachments can be applied to determine the effects of increasing the resistance at R_2 (Figure 6).

Deciding which WHOLE-PART relation applies to which change cannot be determined on the basis of the equations alone. Additional information is required about the values that have remained the same in the circuit. For example, changing a PART has one of two consequences, depending upon whether the other PARTs have remained the same (A3) or the WHOLE has remained the same (A4). The prediction that an increase in R_2 (PART)

Slots: WHOLE, PART1, PART2, ...PARTn

Procedural Attachments:

A1. FIND-WHOLE

A2. FIND-PART

A3. P--> W: CHANGE-PART --> CHANGE-WHOLE
Prerequisites:

(PART1 hasprop Inc(/Dec/Same))

(PART2, ...PARTn hasprop Same)

Consequence:

(WHOLE hasprop Inc(/Dec/Same))

Performance:

(ADJUST WHOLE)

A4. Ps--> P: CHANGE-PART(S) --> CHANGE-PART
Prerequisites:

(PART2, ...PARTn hasprop Inc(/Dec))

(WHOLE hasprop Same/Dec(/Inc))

Consequence:

((PART1 hasprop Dec(/Inc))

Performance:

(ADJUST PART1)

A5. W--> P: CHANGE-WHOLE --> CHANGE-PART
Prerequisites:

(WHOLE hasprop Inc(/Dec/Same))

(PART2, ...PARTn hasprop Same)

Consequence:

(PART1 hasprop Inc(/Dec/Same))

Performance:

(ADJUST PART1)

A6. W--> Ps: CHANGE-WHOLE --> CHANGE-PARTs
Prerequisites:

(WHOLE hasprop Inc(/Dec))

(ABSENCE (PART1, ...PARTn hasprop SAME))

Consequence:

(PART1, ...PARTn hasprop Inc(/Dec))

Performance:

(ADJUST PART1, ...PARTn)

Figure 7. Additional procedural attachments to the WHOLE-PARTS schema.

results in an increase in R_t (WHOLE) depends upon the additional information that the resistance values of the other resistors have remained the same. This information is not given in the problem statement and is not constrained by the equation $R_t=R_1+R_2+R_3$. The equation that describes the relation between total voltage and voltage drops has the identical structure: $E_t=E_{R_1}+E_{R_2}+E_{R_3}$. In this case, however, a decrease in E_{R_1} and E_{R_3} (PARTs) results in an increase in E_{R_2} (PART), not a decrease in E_t (WHOLE).

The same additional information is required to predict qualitative relations between WHOLEs and PARTs described by the formula I=E/R. Although a decrease in I_{R_1} (PART) results in a decrease in E_{R_1} (WHOLE) because the resistance of R_1 remained the same, an increase in R_t (PART) results in a decrease in I_t (PART) because, in this case, the WHOLE, E_t, remained the same. Thus, information about which values have remained the same is critical for constraining the effects of changes. Obtaining this information requires some semantic knowledge about circuits that was not required for quantitative problems. Specifically, batteries and resistors are physical objects that have intrinsic values. Changing the battery has no effect on the value of any resistor; changing a resistor has no effect on the values of the other resistors and no effect on the value of the battery. In other words, total voltage can be changed only by physically changing the battery; a resistance value can be changed only by physically changing, adding, or removing a resistor (barring opens and shorts).

This additional knowledge has been included in the model in the form of procedures for making inferences about values that must remain the same unless explicitly changed. If the resistance value of a resistor has changed, the model infers that the resistance value of the other resistors has stayed the same and the total voltage has stayed the same. If the total voltage has changed, the model infers that the resistance values of all the resistors has stayed the same. These inferences are included in the model's initial representation and are available to the model's procedures for qualitative reasoning.

Consider how each version is applied to reason about the effects of an increase in R_2 in the circuit in Figure 6. The model's initial representation of the problem, including the inferences about which values have remained the same, is shown in Figure 8.

The first version of the model propagates the constraints between the components in this representation by iterating through the following four-step procedure:

1. Select an equation that contains the value that has changed.

2. Instantiate the WHOLE-PARTs schema to interpret this equation.

3. Check each procedural attachment to determine if its prerequisites are satisfied.

4. For every procedural attachment whose prerequisites are satisfied, INFER the change(s) specified in the consequence and add this information to the problem representation.

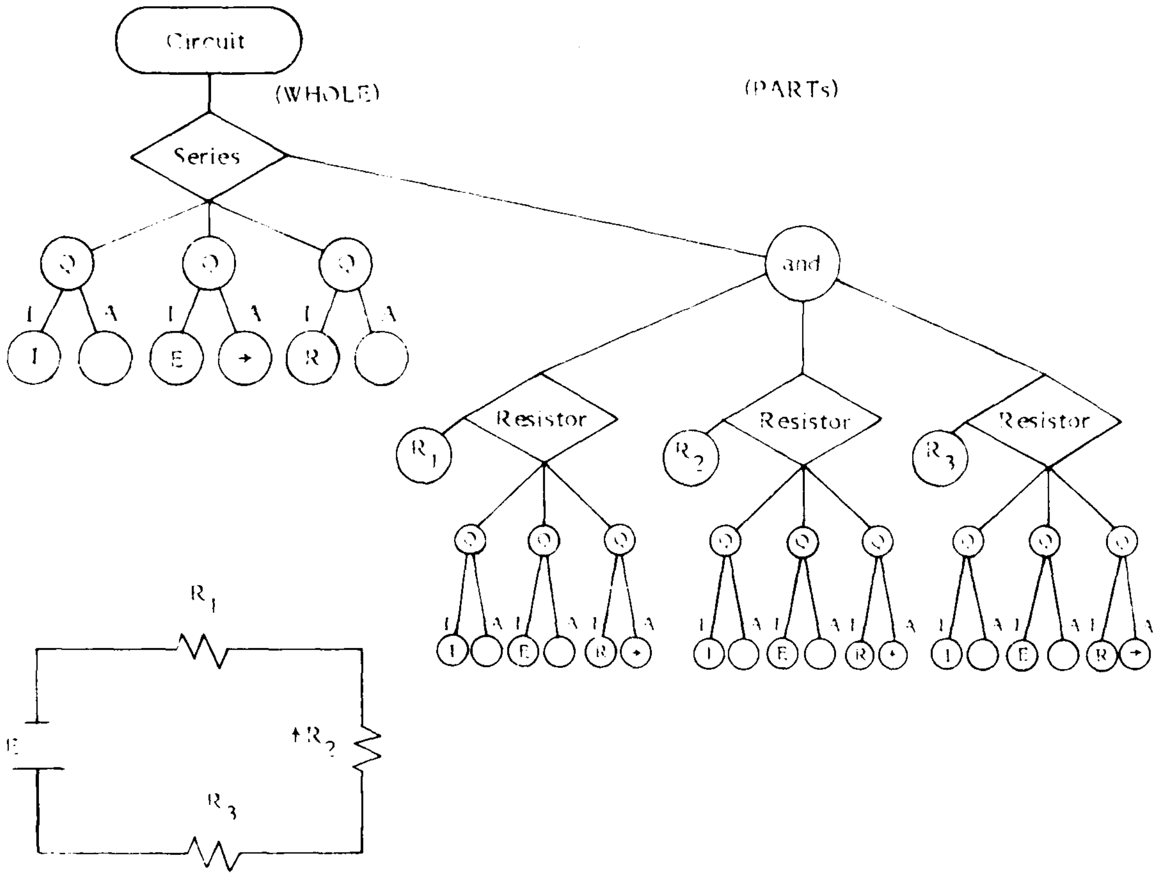

Figure 8. Model's initial representation of problem shown in Figure 6.

The same four steps are then repeated, using the inferred change(s) to propagate additional changes, etc. until the changes in the required circuit values have been constrained. Figure 9 shows how the model applies this strategy to reason about the effects of an increase in R_2. On the first iteration, the model reasons about the changes that can be inferred directly from an increase in R_2. The model selects from its circuit equations (see Table 2) all those that contain the R_2--in this case, EQ1 ($I_{R_2}=E_2/R_2$) and EQ5 ($R_t=R_1+R_2+R_3$). The schematizing productions in the model's WHOLE-PARTS schema instantiate the WHOLEs and PARTs in each equation: (EQ1) E_{R_2} is the WHOLE, I_{R_2} and R_2 are PARTs; (EQ5) R_t is the WHOLE, R_1, R_2, and R_3 are PARTs.

(R_2 hasprop Inc)

SELECT	EQ1: $R_2 = E_{R_2}/R_2$, EQ5: $R_t = R_1 + R_2 + R_3$
INSTANTIATE	EQ1: E_{R_2}=WHOLE, I_{R_2}=PART1, R_2=PART2
	EQ5: R_t=WHOLE, R_1=PART1, R_2=PART2, R_3=PART3
CHECK	A3(EQ5): (PART2 hasprop (Inc) (PART1, PART3, hasprop Same)
INFER	A3(EQ5): (WHOLE hasprop Inc)

(R_t hasprop (Inc)

SELECT	EQ1: $I_t = E_t/R_t$
INSTANTIATE	EQ1: E_t=WHOLE, I_t=PART1, R_t=PART2
CHECK	A4(EQ1): (PART1 hasprop Inc) (WHOLE hasprop Same)
INFER	A4(EQ1): (PART2 hasprop Dec)

(I_t hasprop Dec)

SELECT	EQ3: $I_t = I_{R_1} = I_{R_2} = I_{R_3}$
INSTANTIATE	EQ3: I_t=WHOLE, I_{R_1}=PART1, I_{R_2}=PART2, I_{R_3}=PART3
CHECK	A6: (WHOLE hasprop Dec) (Absence(PART1, PART2, PART3 hasprop Same))
INFER	A6: (PART1, PART2, PART3 hasprop Dec)

(I_{R_1} hasprop Dec)

SELECT	EQ1: $I_{R_1} = E_{R_1}/R_1$
INSTANTIATE	EQ1: E_{R_1}=WHOLE, I_{R_1}=PART1, R_1=PART2
CHECK	A3: (PART1 hasprop Dec) (PART2 hasprop Same)
INFER	A3: (WHOLE hasprop Dec)

(I_{R_2} hasprop Dec)

SELECT	EQ1: $I_{R_2} = E_{R_2}/R_2$
INSTANTIATE	EQ1: E_{R_2}=WHOLE, I_{R_2}=PART1, I_{R_2}=PART2
CHECK	(FAIL)

(I_{R_3} hasprop Dec)

SELECT	EQ1: $I_{R_3} = E_{R_3}/R_3$
INSTANTIATE	EQ1: E_{R_3}=WHOLE, I_{R_3}=PART1, I_{R_3}=PART2
CHECK	A3: (PART1 hasprop Dec) (PART2 hasprop Same)
INFER	A3: (WHOLE hasprop Dec)

(E_{R_1} hasprop Dec)

SELECT	EQ4: $E_t = E_{R_1} + E_{R_2} + E_{R_3}$
INSTANTIATE	EQ4: (E_t=WHOLE) ($E_{R_{1,2,3,}}$ hasprop PART1,2,3)
CHECK	A4: (PART1, PART3 hasprop Dec) (WHOLE hasprop Same)
INFER	A4: (PART2 hasprop Inc)

Figure 9. Reasoning about the effects of an increase in R_2.

The model checks each procedural attachment in Figure 7 to determine if its prerequisites are satisfied in its problem representation. For EQ1, a PART has changed, but the prerequisites of neither A3 or A4 are satisfied, because there is no information in the model's problem representation about whether the WHOLE (E_{R_2}) or the other PART (I_{R_2}) has remained the same. However, for EQ5, the information that the other PARTs, R_1 and R_3, have remained the same is specified and the prerequisites of A3 are satisfied. The model infers that the WHOLE (R_t) has increased and adds this information to its problem representation.

Next the model iterates through the same four steps, using R_t as the value that has changed. EQ1 is the only equation selected because EQ5 was already applied to reason about R_t. After instantiating the WHOLE-PARTS schema, the model notices that the prerequisites of A4 are satisfied because a PART (R_t) has changed but the WHOLE (E_t) stayed the same; therefore, the other PART (I_t) had to change in the opposite direction. The model infers that I_t has decreased and then repeats the same four steps using I_t as the value that has changed.

The rest of the solution continues the iterative process of using each new change to propagate additional changes until all circuit values have been constrained. On cycle 3, the decrease in the total current (WHOLE) is used to infer that the current through each of the resistors (PARTs) has decreased. Cycles 4 and 5 concern the voltage drops across R_1 and R_3. If the current (PART) decreases and the resistance (PART) has stayed the same, then the voltage drop (WHOLE) has decreased. The same inference does not apply to the voltage drop across R_2 because the current (PART) and the resistance (PART) have changed in the opposite direction. However, on cycle 6, the model selects EQ4 ($E_t=E_{R_1}+E_{R_2}+E_{R_3}$) and applies A4 to reason that, if both E_{R_1} (PART) and E_{R_3} (PART) have decreased but the total E_t (WHOLE) has stayed the same, the E_{R_2} (PART) must have increased.

To summarize, in the first version, the input to each set of four steps in Figure 9 is a change in one or more circuit values. The four steps make an explicit connection between this change and the model's knowledge of equations and WHOLE-PART relations. The output of the four-step cycle is an inference about the additional change(s) in circuit values.

The first version works from the generally applicable equations and then has some separate knowledge about how to use them to do qualitative problems. The second version has the separation between the equations and the knowledge about how to use them compiled out. Thus, a single production provides knowledge about the equation and how to apply it. The second version circumvents the four-step reasoning process by connecting each input directly to the corresponding output through the set of productions shown in Figure 10. The implicit connection to the four-step reasoning process is indicated by the equation and procedural attachments shown with each production but this information is not included in the model.

[illegible]

[illegible]

[illegible] R_{p} [illegible]

[illegible]

[illegible]

The first three productions in Figure 10 apply to all circuits and specify the conditions under which a change in quantity directly implies a change in another quantity within the same cluster of quantity nodes corresponding to a single series, parallel, or resistor node. The remaining productions apply to either series or parallel relations and specify the conditions under which a change directly implies a change in one or more quantities in another cluster. These ten productions are sufficient to solve all the qualitative problems in the course.

Discussion and Conclusions

Quantitative Problems

The theoretical analysis showed that, to solve quantitative problems, students needed to know (1) procedures for parsing circuit schematics into representations of the series and/or parallel relations between resistors, (2) the circuit equations listed in Table 2, (3) a backward search strategy for selecting equations, and (4) a WHOLE-PARTs schema for identifying the relations between the quantities in equations. In the model, the absence of anyone of these components of knowledge would lead to predictable errors. Removing all or part of the model's parsing procedure would result in an absent or incorrect problem representation. The model would have no way to distinguish series from parallel resistors and inappropriately select, for example, a series equation to solve for a value in a parallel circuit. Removing the model's equation or backward search strategy would cause the model to stop after representing the problem, and the lack of a WHOLE-PARTs schema would lead to wrong operation errors or failure to solve an equation.

If subjects make the same errors, this may be taken as evidence that they lack the corresponding components of knowledge. It is unlikely that subjects would lack an equation or fail to solve it once selected because they can refer to the equations at all times and the operations required to solve the equations are simple. However, students may lack a backward search strategy, as this is not taught explicitly in the course. For example, in BE/E, students are told that, to solve for the voltage drop across R_1 in Figure 1, they would (1) infer that the current at R_2 is 2 A, (2) solve for the voltage drop across R_2 by multiplying the 2 A by the 10 ohms resistance, and (3) solve for the voltage drop across R_1 by subtracting the 20 volts from the total voltage to get 40 volts. This sequence of steps corresponds to the model's solution but, as shown in Table 1, solving for the current at R_2 was preceded by 12 cycles of planning. This is not taught explicitly in the course. The same is true for all the problems--subjects are shown the actual steps required to solve the problem but not the planning underlying those steps.

However, the lack of a backward search strategy need not lead to incorrect solutions, just less efficient ones on some problems. That is, although the model's backward search strategy produces the solution with the least number of steps, these problems can be solved by using a "solve for anything" forward search strategy that requires no planning. Since all circuit values are mutually constrained, this strategy would eventually result in the correct solution on all problems. In fact, it would lead to a solution identical to the model's backward search solution on problems in which all the required variables are given in the problem statement and the first equation selected can be solved directly.

If subjects do use a backward search strategy, it is possible that problem difficulty may vary as a function of the maximum number of goals on the stack. One argument against this hypothesis is that, except on rare occasions, no problem solution requires stacking more than four goals. Most problems require only one or two. Evidence suggests that even young children can stack this many goals (Klahr & Robinson, 1981).

Finally, difficulties on quantitative problems may also result from attempts to relate circuit equations to the "moving crowd" analogy (Gentner & Gentner, 1981) used to teach students about circuit relations in the BE/E course. The moving-crowd analogy consists of a bunch of little objects (balls) that are given a push to start them rolling along a path that varies in smoothness. Current corresponds to the rate at which the objects move along the path, applied voltage corresponds to the push applied to start the objects in motion, resistance corresponds to the greater friction of the rough spots relative to the rest of the path, and voltage drops correspond to the amount of force given up by the objects across the rough spots. Some of the relations between the objects in this analogy are inconsistent with the relations between the quantities in equations; thus, students may become confused.

In a series circuit, current is the same through all resistors ($I_t=I_{R_1}=...I_{R_n}$). However, in the analogy, there is no constraint that the objects move at the same rate at all points along the path. On the contrary, they should gradually slow down because they lose force each time they encounter friction. This, in turn, implies that the applied force is not distributed proportionally across all the points of friction. Objects give up proportionally more force across rough patches earlier in the path because they are moving faster. In fact, if the friction at an initial point in the path were large enough, the analogy predicts that all the force is lost at that point, corresponding to dropping the total applied voltage across the first resistor in a series circuit. On the other hand, if the path were almost "frictionless," the analogy predicts that some of the force might be left over, which corresponds to not distributing the full amount of applied voltage and violates the constraint that $E_t=E_{R_1}+...E_{R_n}$. Thus, if students apply the moving crowd analogy to reason about circuit values, they may not realize, for example, that, if given the current at one point in a series circuit, they can infer that current is the same at all other points in the circuit. Further, if they are told that current is the same, they may be confused as to why.

In summary, the theoretical analysis predicts that subjects should have little difficulty parsing circuits, or remembering, selecting, and solving circuit equations. However, they may have difficulty because of the lack of a backward search strategy or confusion in applying the moving-crowd analogy to reason about circuit relations.

Qualitative Problems

Two alternative versions of a constraint propagation model of qualitative reasoning were presented. In the first version, the constraints were related explicitly to the circuit equations and to the WHOLE-PART relations between those quantities. In the second version, the connection between the qualitative reasoning rules and circuit equations and WHOLE-PART relations is implicit.

The BE/E instruction on qualitative problems explicitly presents the knowledge included in the second version of the model. First students are told that applied voltage

and resistance values correspond to actual physical objects like batteries and resistors; therefore, changing the applied voltage has no effect on any of the resistance values, and changing a resistor has no effect on the applied voltage or any of the other resistance values. Students also receive hands-on experience in changing batteries and resistors and determining the effects of these changes. Then students are shown solutions to qualitative problems that are identical to the solution steps generated by the model. In additional, each solution step is explicitly connected to the corresponding circuit equation--this connection was left implicit in the second version of the model. Thus, statements like, "IF current increases and resistance remains the same, THEN the voltage drop increases" were accompanied by, and often replaced by, solution steps expressed as $I^{\wedge} \approx E?/R >$.

It is possible that subjects will have no difficulty learning to solve qualitative problems since the course explicitly teaches the minimum knowledge required to solve these problems. Further, since the effects of most circuit changes are easily summarized, students may be encouraged to memorize general rules for solving qualitative problems, without understanding the underlying reasoning, especially since they are introduced to qualitative problems in blocks of the same type of problems. In this case, students would encounter predictable difficulty on problems that cannot be solved using general rules.

Also, the lack of an explicit connection between solution steps and circuit constraints may result in additional generalizations that work for most problems, but lead to predictable errors on others. Specifically, the following three rules correctly summarize most of the relationships within or between any cluster of circuits values:

1. If resistance changes within a cluster, then current changes in the opposite direction.

2. If the voltage within a cluster changes, resistance remains the same.

3. For all other relations, WHOLEs and PARTs change in the same direction.

Rule 1 is always correct. The only exception to Rule 2 occurs when a resistance value has changed within the cluster. This is of no consequence, however, since Rule 2 would never be applied to reason about the resistance value that has changed--such information is always given in the problem statement. Except for the errors described below, Rule 3 also leads to correct predictions most of the time. On 82 percent (53/65) of the cycles, the correct inference is that a WHOLEs and its PARTs have changed in the same direction--if the model's inferences about the effects of resistance on current (Rule 1) are not included, then the total rises to 90 percent (53/59). In the course, the total is higher since the distribution of problem types is not balanced and includes more problems in which Rule 3 is always correct.

The implications of the WHOLE-PARTs generalization in Rule 3 are shown in the productions in Figure 11. The first three productions concern the implications of extending the WHOLE-PARTs generalization to reason about within-cluster relations (i.e., the effects of changing a total value on another total value, or the effects of changing one value associated with a resistor on another value associated with the same resistor):

1. E > I infers that, if the voltage value within a cluster changes, then current changes in the same direction.

2. I > E infers that, if current changes, then within-cluster voltage changes.

		Within-cluster
$E > I$.	IF	(E hasprop Inc(/Dec))
	THEN	(I hasprop Inc(/Dec))
	A,EQ	A7(W--> P), I=E/R
	ERROR	Series and Combination R_n Inc(/Dec): E_{R_n} Inc(/Dec) BUT I_{R_n} Dec(Inc)
$I > E$.	IF	(I hasprop Inc(/Dec))
	THEN	(E hasprop Inc(/Dec))
	A,EQ	A5(P--> W), I=E/R
	ERROR	All Circuits R_t Inc(/Dec): I_t Dec(Inc) BUT E_t Same Series and Combination R_n Inc(/Dec): I_{R_n} Dec(/Inc) BUT E_{R_n} Inc(Dec) Parallel R_{B_n} Inc(/Dec): I_{B_n} Dec(/Inc) BUT E_{B_n} Same
$R > E$.	IF	(R hasprop Inc(/Dec))
	THEN	(E hasprop Inc(/Dec))
	A,EQ	A6(P--> W), I-E/R
	ERROR	All Circuits R_t Inc(/Dec): E_t Same Parallel R_{B_n} Inc(/Dec) BUT E_{B_n} Same
		Between-cluster
$I_t <> I_{R_n}$.	IF	(I_t hasprop Inc(/Dec)
	THEN	(I_{R_n} hasprop Inc(/Dec)
	A,EQ	A8(W--> Ps), I_t I_{R_1} ...I_{R_n}, I_{Req} I_{B_1} +...I_{B_n}
	ERROR	Parallel R_{B_1} Inc(/Dec): I_t Dec(/Inc) BUT I_{B_n} Same
$E_t <> E_{R_n}$.	IF	(E_t hasprop Inc(/Dec))
	THEN	(E_{R_n} hasprop Inc(/Dec))
	A,EQ	A8(W--> Ps), $E_t = E_{R_1} + \ldots E_{R_n}$, $E_{Req} = E_{B_1} \ldots E_{B_n}$
	ERROR	Series and Combination R_1 Inc(/Dec): E_t Same BUT E_{R_1} Inc(/Dec), E_{R_n} Dec(/Inc) Combination R_3 Inc(/Dec): E_{B_1} Inc(/Dec BUT E_{R_2} Dec(/Inc)
$R_n <> R_t$.	IF	(R_n hasprop Inc, Added(/Dec,Removed))
	THEN	(R_t hasprop Inc(/Dec))
	A,EQ	A5(P--> W), $R_t = R_1 + \ldots R_n$, Req $1/((1/R_{B_1}) + \ldots (1/R_{B_n}))$
	ERROR	Parallel R_n Added(/Removed) BUT R_t Dec(/Inc)

Figure 11. Overgeneralized productions for qualitative reasoning.

3. R > E infers that, if the resistance within a cluster changes, then voltage within the cluster changes in the same direction.

The only relations that are not correctly predicted by each production are indicated by ERROR in Figure 11:

1. E > I--When a change in a voltage drop corresponds to an increase in the resistance value within the same cluster, current changes in the opposite direction.

2. I > E--When an increase in current is associated with a decrease in resistance with the same cluster, voltage remains the same or changes in the opposite direction.

3. R > E--When total resistance changes, or when the resistance value of a branch in a simple parallel circuit changes, voltage within the corresponding cluster remains the same.

The next three productions in Figure 11 predict the between-cluster effects of changing circuit values: A WHOLE (I_t, E_t, or R_t) and its corresponding between-cluster PARTs (I_{R_n}, E_{R_n}, or R_n respectively) change in the same direction. The following errors are associated with these between-cluster productions:

1. $I_t < < I_{R_n}$--In a parallel circuit, when the change in total current results from a change in the resistance value of one of the branches, the current in the other branches remains the same.

2. $E_t < > E_{R_n}$--In a combination circuit, when a parallel branch contains two or more series resistors and one of the resistance values is changed, then total voltage of the branch changes in the opposite direction of the voltage drop across the resistor that has not changed.

3. $R_n < > R_t$--In a simple parallel circuit, when a resistor is added or removed, then *total resistance changes in the opposite direction predicted by production* $R_n < > R_t$.

A final difficulty on qualitative problems may result from student attempts to apply the moving-crowd analogy to reason about qualitative changes in circuit values. Although the analogy is consistent within many circuit relations, it leads to some predictable errors.

Referring back to the productions in Figure 10, the moving-crowd analogy correctly predicts the within-cluster relations between current, voltage, and resistance. If the amount of friction stays the same, then increasing the force applied to the objects--or the amount of force they give up across a rough patch--increases their rate of movement. By analogy, increasing the applied voltage or a voltage drop should increase electrical current, which corresponds to the first production (P1) in Figure 10.

The relations predicted by productions P2 and P3 are also constrained by the moving-crowd analogy. If the objects' rate of movement increases and the amount of friction remains the same, then the objects give up more force going across a rough patch. By

analogy, if current increases through a resistor and resistance stays the same, then voltage drop across that resistor increases (P2). Also, if the total amount of friction along the path increases and the amount of force applied to the objects remains the same, then the rate of the objects decreases. By analogy, increasing the total resistance decreases the current (P3).

The relations in the moving-crowd analogy are also consistent with most between-cluster relations that involve either the effects of a WHOLE on all its PARTS (P4 and P7 in Figure 10) or the effect of a PART on a WHOLE (P8 and P9). For example, increasing the initial rate of the objects increases their rate along the rest of the path. By analogy, increasing the total current increases the current through all resistors in series with the total (P4). Increasing the amount of push applied to the objects increases their rate and, therefore, the amount of force they lose across the rough spots. By analogy, increasing the voltage applied to a parallel circuit increases the voltage drops across all the branches (P7). Increasing the friction of a rough spot or adding another rough spot increases the total amount of friction along the path. By analogy, increasing the resistance value of a resistor or adding on another resistor in series increases the total resistance (P8 and P9). However, the effects of adding on a resistor in parallel are not obviously constrained by the analogy. If the resistance value of the new resistor is smaller than the resistance value of an existing branch, then the average rate of the objects might be predicted to increase, which would be consistent with P10. On the other hand, if the resistance value of the new resistor were greater, the average rate of the objects would probably be predicted to decrease.

The essential, time-dependent reasoning encouraged by the moving-crowd analogy *does not constrain the effects* of PARTs on another PART of the circuit, as specified by productions P5 and P6. P5 constrains the WHOLE-PARTs relations between current values in a parallel circuit; P6 constrains the WHOLE-PARTs relations between voltage values in series circuits.

P5 and P6 were used to predict the effects of an increase in R_3 in the combination circuit in Figure 12. An increase in R_3 increases the total resistance and decreases the total current and the current through R_1 and Req. The voltage drop across R_1 also decreases, but the total voltage remains the same. Therefore, the voltage drop across Req must increase (P6). If E_{Req} increases, then the voltage drops across each parallel branch increases. The current through the second branch increases, but the total current decreases; therefore, the current through the first branch decreases (P5).

These implications of increasing R_3 are not constrained by the moving-crowd analogy. Envisioning a bunch of little objects moving along a path predicts that changing a resistance value should not affect any values "prior" to the change. That is, a change in R_3 could not affect any values at R_1 and R_2 because the objects have already passed through these resistors by the time they encounter the change. Also, the objects that travel through the second branch do not encounter the change; therefore, the values in the second branch do not change.

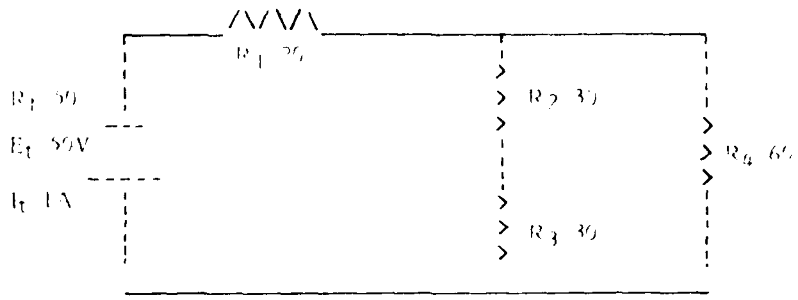

Figure 12. Variational analysis of a combination circuit (adapted from Example 2-7, B/E Module 6-2).

In summary, although the minimum knowledge (Figure 10) for reasoning correctly about qualitative changes in circuit values is explicitly taught in the BE/E course, students may encounter difficulties unless the circuit constraints underlying those rules are also made salient. The difficulties predicted by the theoretical analysis include overextensions of the moving-crowd analogy to reason about circuit relations and overgeneralized productions that provide a shortcut to the repetitious actions involved in most solutions. Both the moving-crowd analogy and the generalized productions lead to correct and efficient solutions to most problems, but result in violations of circuit constraints on others. However, unless these constraints are salient to the student, he/she will have no way of preventing these errors and will be confused about why certain rules sometimes work and sometimes do not.

EXPERIMENT I

Purpose

The purpose of Experiment I was to test the predictions of the theoretical analysis.

Method

Subjects

Subjects were seven males selected because their Armed Services Vocational Aptitude Battery (ASVAB) test scores (Forms 8, 9, and 10) showed that their academic and career histories were similar to those of the BE/E recruit population. Three subjects were juniors in high school, one was a high school sophomore, and three were in their third year of college. None had ever studied basic electricity or was familiar with electric circuits. Navy recruits were not used as subjects mainly because it would have been difficult to (1) arrange protocol sessions with recruits, and (2) conduct any follow-up studies, since most recruits are transferred to "A" schools at other sites when they complete BE/E. Table 3 shows the ASVAB Navy standard profile scores achieved by the seven subjects.

Table 3

ASVAB Navy Standard Profile Scores Achieved by Subjects

ASVAB Test	S1	S2	S3	S4	S5	S6	S7
Numerical operations (NO)	42	46	55	61	50	59	53
Word knowledge (WK)	44	50	43	48	57	60	62
Arithmetic reasoning (AR)	43	42	53	56	63	60	67
Mathematics knowledge (MK)	54	53	53	59	66	59	69
Electronics information (EI)	37	58	54	54	58	61	65
Mechanical comprehension (MC)	45	51	54	58	53	58	65
General science (GS)	44	52	50	59	65	65	63
Paragraph comprehension (PC)	54	52	40	57	60	54	63
Coding speed (CS)	47	53	54	42	54	35	60
Auto shop (AS)	42	53	49	47	47	22	55
Verbal (VE) (average of the WK and PC scores)	47	51	41	51	58	58	63

Note. Subjects 1, 2, and 3 were juniors in high school; subjects 4, 6, and 7 were in their third year of college; and subject 5 was a high school sophomore.

Table 4, which compares these scores to the mean profile scores obtained for BE/E graduates and attrites in previous studies (Dann, 1978; Federico & Landis, 1979), shows that they fell well within the range of abilities found in the BE/E student population. Most subjects had scores that were within one standard deviation (SD) of either the attrite or graduate means; subjects 1, 2, and 3 generally scored closer to the attrite mean; and subjects 5, 6, and 7, to the graduate mean.

Materials

Materials were the first six modules of the programmed instruction (PI) version of BE/E. Modules 1 through 4 concerns series circuits, Module 5, parallel circuits; and Module 6, combination series-parallel circuits. Each module consists of two or more lessons that (1) begin with an introduction to quantitative or qualitative circuit relations, (2) show step-by-step solutions to problems involving these relations, and (3) ask the student to solve similar problems. During PI, answers to all the problems are available and students are encouraged to check their own performance. Following PI, self-check progress tests are given on each lesson topic. After students have completed a series of lesson topics, they take a module test that they must pass before beginning the next series. Subjects also learned to solve problems involving opens, shorts, and conversion of electrical units, and received hands-on experience using equipment to solve problems.

Table 4

Comparison of Subjects' ASVAB Scores and Mean ASVAB Profile Scores Obtained for BE/E Graduates and Attrites in Previous Studies

ASVAB[a] Test	S1	S2	S3	S4	S5	S6	S7
Dann (1978) Study							
NO	+A	+A	+GA	+G	+GA	+G	+GA
WK	-A	+A	-A	+A	+GA	+GA	+GA
AR	-A	-A	+A	+GA	+G	+G	-G
MK	+GA	+A	+A	+GA	+G	+GA	-G
EI	-A	+GA	+GA	+GA	+GA	+GA	+G
MC	-A	+GA	+GA	+GA	+GA	+GA	+G
GS	-A	+A	+A	+GA	+G	+G	+GA
Federico & Landis (1979) Study							
NO	+A	+A	+GA	+G	+GA	+G	+GA
WK	-A	+A	-A	-A	+GA	+GA	+GA
AR	-A	-A	+GA	+GA	+G	+GA	+G
MK	+GA	+GA	+GA	+GA	+G	+GA	-G
EI	-A	+GA	+GA	+GA	+GA	+GA	+G
MC	-A	+A	+GA	+GA	+GA	+GA	+G
GS	+A	+A	+A	+GA	+GA	+GA	+GA

+GA--Score is within 1 SD of both graduate and attrite means.
+A--Score is within 1 SD of attrite mean.
+G--Score is within 1 SD of graduate mean.
-A--Score is too low to be within 1 SD of attrite mean.
-G--Score is too high to be within 1 SD of graduate mean.

[a]PC, CS, and AS tests not included because there were no corresponding tests in the ASVAB forms used in the previous studies.

Procedure

About twice a week, each subject was observed individually during protocol sessions that lasted anywhere from 1 to 2 hours. During each session, the subject paced himself through the contents of a BE/E module in the presence of an experimenter trained in taking protocol observations. Subjects had paper and pencil available at all times but were not permitted to use a calculator, in accordance with the Navy course regulations. Each session was tape-recorded and the subject was encouraged to think aloud while reading through the module and solving the problems. If a subject neglected to comment on how he had solved a particular problem, the experimenter prompted the subject with specific questions and/or described in detail the sequence of equations the subject had written down during problem solution. If a subject failed to respond to a problem or responded incorrectly, the experimenter showed the subject the correct solution, as done in BE/E.

Analyses

Transcriptions of the verbal protocols were analyzed to determine (1) the percentage of correct responses on quantitative and qualitative problems, and (2) the actual type of incorrect responses made. A subject's response was judged as incorrect if he gave a wrong answer, failed to give any answer, or answered correctly only after extensive prompting by the experimenter.

Results

Quantitative Problems

Subjects' performance on quantitative problems indicated that students used (1) procedures for parsing circuit schematics into representations of the series and/or parallel relations between resistors, (2) the necessary circuit equations, (3) either a forward or backward search strategy for selecting equations, and (4) a WHOLE-PARTS schema for identifying the operation to solve an equation. Evidence showed that failure was caused because students failed to understand circuit constraints. This deficiency caused them to (1) overextend the moving-crowd analogy and (2) relax the conditions for selecting variables to map into equations.

The most direct evidence that subjects used procedures for parsing circuit schematics came from their verbal descriptions of the series and/or parallel relations between resistors. For example, in solving the problem in Figure 13, subjects referred to R_2 and R_3 as "parallel resistors," the combined resistance of R_2 and R_3 as "equivalent resistance," and R_1, Req, and R_4 as "being in series." Other evidence suggested that subjects did not make many errors in parsing circuits because (1) the schematics in the BE/E course are relatively simple, and (2) subjects knew that all of the circuits within any module or test were of the same type (i.e., series, parallel, or combination).

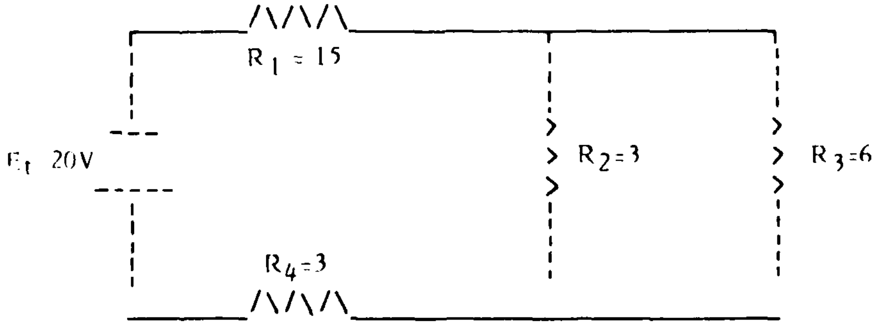

Figure 13. Solving for I_{R_2} in a combination circuit (adapted from Figure 1-13, BE/E Module 6-1).

Evidence that problem difficulty was not due to lack of circuits equations or WHOLE-PARTS schema came from the fact that subjects had the equations available at all times and, for the most part, had no difficulty determining the operation required to solve an equation once all the required variables had been determined.

Evidence that students had an adequate strategy for selecting equations came from their ability to solve many of the three and four-goal problems, as indicated by the proportion of correct responses percentages correct shown in Table 5. Further, the sequence in which subjects selected equations and their accompanying verbal statements frequently reflected the use of a backward search strategy, as illustrated by the following protocol excerpt. When one subject was given the total current and the individual resistance values and asked to solve for the total power, he said:

> Power equals...I times E, right?
> (Writes: P = IxE)
> Okay, so we have I, and I equals 3 amps. Now to get E, it's gonna be
> E equals R times I.
> (Writes: E = RxI)
> (Adds individual resistance values.)
> R equals 60 ohms. Three times 60, which is 90 volts...180 volts.
> (Returns to first equation.)
> So that'd be 3 times 180... 540 watts.

Thus, any difficulty that students had in solving the problems in Table 5 cannot be attributed to the lack of parsing procedures, circuit equations, or a WHOLE-PARTS schema. The results presented below present evidence that the difficulty resulted from a lack of understanding of circuit constraints.

Table 6 summarizes the types of errors subjects made on the quantitative problems. As shown, 25 percent of the errors in series circuit problems resulted from attempts to apply the moving-crowd analogy to reason about circuit values. An examination of these errors showed that the moving-crowd analogy caused subjects to envision that current becomes increasingly slower as it flows through the circuit because (1) each successive resistor slows the current flow, (2) voltage is being dropped at each successive resistor, and (3) voltage is what pushes current. For example, when one subject was given the current through the first resistor in a series circuit and asked to determine the current through the second resistor, he said:

> It (I_{R_2}) will be less. Okay. It'll be different than this one (I_{R_1}) because there's a resistor. Because, right here (R_1), there's 15 volts going through...and it's being decreased by 5 ohms. So it (I_{R_2}) should be different.

Another subject also predicted that current would be less through R_2. When told that it was the same, he said:

> But current is decreased when you throw these things in...resistors. Current is determined by ohm's law, and ohm's law says, "The more resistance you got, the less the current."

Table 5

Percentage of Correct Responses on Quantitative Problems by Problem Type and Number of Goals on Stack

Type of Circuit Problem	Goals on Stack				
	1 (%)	2 (%)	3 (%)	4 (%)	7 (%)
Series:					
E_{R_n}	100(2)	86(1)[a]	29(2)[a,b]	100(1)[a,b]	--
I	81(3)	--	--	--	--
I	100(3)	100(7)	--	--	--
E	100(2)	--	--	--	--
E	100(3)	86(3)[a]	--	--	--
R	81(3)	--	--	--	--
P	100(1)	--	86(3)	--	--
Parallel:					
I	90(3)	--	95(3)	--	--
E_{R_n}	90(3)	--	--	--	--
R(X/+)	100(4)	--	--	--	--
R(R/N)	100(4)	--	--	--	--
R(1/)	100(6)	--	--	--	--
P	--	--	86(1)	93(2)	--
Combination:					
I_{R_2}	--	--	--	--	29(1)[b]
R_3	--	--	29(1)[b]	--	--
P	100(1)	--	93(2)	--	--
R_t	--	86(1)	--	--	--
I_t	71(1)	--	--	--	--
I_{R_2}	--	--	--	29(1)[b]	--

Note. Number in parentheses beside each percentage indicates the number of problems on which the percentage is based.

[a]Errors caused by failure to distinguish between voltage drops and resistance (E=R).

[b]Errors caused by failure to distinguish between local and global values (LG).

Table 6

Types of Errors Made on Quantitative Problems

Type of Errors	Type of Circuit Problem: Series (N=48) (%)	Parallel (N=8) (%)	Combination (N=22) (%)
1. Resulted from attempts to apply the moving-crowd analogy to reason about circuit values.	25	0	0
2. Resulted from failure to distinguish between voltage drops and resistance (E=R).	35	0	0
3. Resulted from failure to distinguish between local and global variables when using ohms' law (LG).	29	0	50
4. Resulted from failure to answer.	8	87	36
5. Other.	3	13	14
Total	100	100	100

Note. When multiple errors occurred in a single problem, each was counted separately.

The next error listed in Table 6 resulted from subjects' failure to distinguish between voltage drops and resistance (E=R). For example, four of the seven subjects solved for the voltage drop across R_1 in Figure 1 by subtracting the resistance of R_2 from the total voltage. Another subject solved for the total resistance, subtracted the value of R_1 from the total, and answered, "20." When informed that the correct answer was 40 volts, not 20 ohms, he commented:

> Two equals...60 over the resistance. So why isn't it 20? To get R, which is 10 plus X. And E is what E was, 60, and that (I) is 2. And 2 equals 60 over 30. So to get 30, that has to be 20--not 40.
>
> (Later he said)
>
> Oh, that's not a resistance drop, that's R_2.

Subjects' use of terms like "resistance drop" (above example) and "2-volt resistor," and statements like, "20 ohms resistance indicates voltage drop...because it's a resistor," suggests that one possible source of the E=R error was a semantic confusion between voltage drops and resistance; that is, voltage drops always occur across resistors and the amount of voltage dropped is proportional to the amount of resistance.

This confusion might have been further encouraged by the fact that the voltage drop and resistance values within a cluster are identical when the current in a series circuit is 1 amp. For example, in one series problem, the value of the total voltage and all but one of the three resistance values were given, and the task was to solve for the voltage drop across the one resistor. Since the voltage drops and resistance values were identical, one subject concluded, "All you have to do, then, is look at how many volts there are." Another subject commented, "There's three resistors...equal to the voltage. So you just add up R_1 and R_2 and match it by voltage."

The third type of error in Table 6 involved missing local and global variables when using ohm's law (LG). For example, the problem in Figure 1 is to determine the voltage drop across R_1, but neither the resistance of R_1 nor the current through R_1 are given. One subject attempted to solve for the resistance at R_1 by dividing the total voltage by the current. Another divided the total voltage by the resistance at R_2 to determine the current through R_1. Other examples of LG errors occurred on problems involving combination circuits. For example, to solve the voltage drop across R_2 in Figure 13, four of the seven subjects divided the total voltage by the resistance of R_2.

The pattern of errors in Table 6 suggests that subjects stopped applying the moving-crowd analogy once they realized it led to incorrect conclusions about circuit relations in series circuits. Also, it appears that subjects confused voltage drops and resistance values only during the early modules on series circuits but continued to confuse local and global variables.

Qualitative Problems

The results of students' performance on qualitative problems also supported the predictions of the theoretical analysis. Although the minimum components of knowledge required to solve these problems had been presented explicitly in the BE/E course, subjects still had difficulty. The types of errors they made indicated that they failed to understand the constraints underlying problem solutions and reasoned using overgeneralized rules that corresponded to the productions shown in Figure 11. These rules omitted tests for conditions that must remain the same for a given change to have the predicted effect and therefore violated circuit constraints when these conditions were not satisfied. However, as with the quantitative problems, constraint violations were not detected by the majority of problems in the BE/E course.

As shown in Table 7, the percentage of correct solutions on the majority of qualitative problems was quite high. Although it appears that subjects had no difficulty in solving qualitative problems, a closer examination of the problems in the course cautioned against this interpretation. For one thing, more often than not, the multiple choices listed for a problem precluded an error that might have occurred. For example, for the seven problems requiring subjects to reason about the effects of changing R_2, R_3, or R_4

Table 7

Percentage of Correct Responses by Problem Type and Difficulty Level (Qualitative Problems)

Change[a]	Predicted Effects I_t (%)	I_{R_n} (%)	E_t (%)	E_{R_N} (%)	R_t (%)	R_n (%)
Series:						
E_t	100(12)	----	----	100(3)	90(3)	----
R_t	100(4)	----	82(4)	----	----	----
R_n	100(8)	----	100(1)	43(2)	100(1)	----
Parallel:						
E_t	100(13)	100(3)	----	100(3)	93(4)	100(2)
R_n	100(15)	88(6)	88(6)	91(8)	94(7)	100(5)
Combination[b]						
E_t	----	----	100(6)	----	----	----
R_1	----	----	100(2)	----	----	----
$R_{2,3,4}$	----	----	88(7)	----	----	----

Note. The number in parentheses behind each percentage indicates the number of problems in each type.

[a]Indicates the value that has changed.

[b]A single percentage is listed for each type of combination problem because all were multiple-choice and each choice grouped a set of effects. This made it impossible to judge subjects' predicted effects.

(see Figure 12), the percentage of correct responses was 88. However, all seven problems were multiple-choice format and five could have been answered correctly by simply eliminating obvious distractors that violated constraints (e.g., the currents through two series resistors changing in opposite directions or the voltage drop across two parallel branches changing in opposite directions). Only one of the seven problems included a choice that corresponded to one of the common errors made by the subjects; this is the problem on which the errors occurred and subjects chose the distractor.

Even in problems that did not have a multiple-choice format, the sequencing of questions often seemed to preclude errors. For example, one of the errors predicted for parallel problems was that subjects might reason that, if the resistance of a parallel branch increases, then total current decreases; therefore, the current through all the branches decreases (I_t<>IR_n). This error was more likely to occur if the question focussed subjects' attention on the change in total current by asking them to predict the effects of the change in resistance on (1) total resistance, (2) total current, and (3) branch currents, in that order, without explicitly asking about total voltage and voltage drops. It was less likely to occur if subjects were first required to determine that the total voltage had remained the same. If the total voltage had remained the same, then the voltage drops across each branch would have remained the same; therefore, current would have remained the same. Thus, it appears that the way questions were sequenced within a problem also determined whether a problem was diagnostic.

For these reasons, even when subjects responded correctly, they were frequently asked to solve the problem again by just referring to the circuit schematic and not to the problem statement or the multiple-choices. As predicted, in the absence of multiple-choice and guided responses, subjects did make the errors associated with the over-generalized productions in Figure 11. The distribution and types of errors are shown in Table 8.

The first type of error is associated with the time-dependent envisioning encouraged by the moving-crowd analogy. For example, when asked to predict the effects of an increase in the resistance at R_2 in Figure 6, one subject's time-dependent model of current flow led him to represent R_1 and R_3 as being "prior to" and "subsequent to" the change at R_2, respectively. He therefore concluded that the voltage drop at R_1 must remain the same since the current passed through R_1 before it encountered any changes at R_2:

> But...I was just wondering, you know...okay, if it should be less in R_1, or if it should be less in R_3, or if it should be less in both. Offhand, I would say it would be the same as R_1, and it would be greater in R_2, and it would be less at R_3. This one (R_1) would stay the same. And this one (R_2) would get greater, so this one (R_3) would get less...become less. I was just thinking...that the amount of voltage going into this resistor (R_1)... because it's prior to the change...would be the same. And the amount of voltage going into this resistor (R_3), because it's subsequent to the change, would be less.

Consistent with the findings on the quantitative problems, analogy errors were restricted to reasoning about qualitative changes in series circuits. However, the within- and between-cluster errors predicted by the productions in Figure 11 persisted through combination problems.

Table 8

Types of Errors Made on Qualitative Problems

Type of Error	Type of Circuit Problem: Series (N=25) (%)	Parallel (N=20) (%)	Combination (N=55) (%)
1. Due to application of moving-crowd analogy	12	0	0
2. Due to within-cluster problems:			
a. E>I	0	0	5
b. I>E	32	35	24
*c. R>E	36	25	7
d. Balancing	0	0	11
	68	60	47
3. Due to between-cluster problems:			
a. I_t<>I_{R_n}	0	25	15
**b. E_t<>E_{R_n}	16	0	20
c. R_n<>R_t	0	15	0
d. Balancing	4	0	18
	20	40	53
Total	100	100	100

*IF (R_n hasprop Inc (/Dec/Same))
THEN (E_{R_n} hasprop (Inc/Dec/Same))
A,EQ A6(P-->W), I=E/R
ERROR Series and Combination
R_1 Inc(/Dec): R_n Same BUT E_{R_n} Dec(/Inc)
Parallel
RB_1 Inc(/Dec): EB_1 Same

** IF (E_t hasprop Inc(/Dec/Same))
THEN (E_{R_n} hasprop Inc(/Dec/Same))
A,EQ A8(W-->Ps), $E_t=E_{R_1}+\ldots E_{R_n}$, $E_{Req}=EB_1=\ldots EB_n$
ERROR Series and Combination
R_1 Inc(/Dec): E_t same BUT E_{R_1} Inc(/Dec), E_{R_n} Dec(/Inc)
Combination
R_3 Inc(/Dec): EB_1 Inc(/Dec) BUT ER_2 Dec(/Inc)

The types of within- and between-clusters shown in Table 8 are discussed below. The two productions at the bottom of the table were included to illustrate two minor modifications required to account for subjects' errors.

1. <u>Within-cluster Errors</u>

a. E>I is associated with the production that says, any time the voltage changes, current changes in the same direction. This rule led to correct predictions as long as resistance remained the same but resulted in errors if the change in voltage was associated with a change in resistance. In this study, the E>I error occurred on problems about an increase in resistance within a parallel branch of a combination circuit. Subjects reasoned, for example, that, if the voltage drop across Req increased, then the current through Req must have increased. In fact, the current through Req decreased because total resistance increased.

b. I>E corresponds to the production that says that, any time current changes, the voltage changes in the same direction. This production is true if the current has changed because of an increase in the applied voltage but it leads to predictable errors if the current changed because of a change in resistance.

c. R>E corresponds to the production that says that, if resistance changes, then voltage changes in the same direction. As predicted, some subjects applied this rule to reason about total circuit values and said, for example, that total voltage would increase if total resistance increased. Further, as shown by the modified condition of production R>E in Table 8, some subjects further generalized this rule to include the case in which resistance has remained the same.

d. Balancing was not predicted by the production rules in Figure 11 but is included here since it involves reasoning about changes within a single cluster.These errors occurred on combination problems and involved reasoning that, if two values change in opposite directions, then the changes balance each other out so the third value remains the same.

2. <u>Between-cluster Errors</u>. These errors correspond to the between-cluster productions in Figure 11 that assume that wholes and parts always change in the same direction.

a. It<>IR$_n$ involves the assumption that, if the total current changes, then all the current through all the resistors always changes in the same direction, regardless of the resistance values. This led to the It<>IR$_n$ error when the change in total current resulted from a change in the resistance within a parallel branch of a combination circuit like the one in Figure 11. In this case, the two branch currents change in opposite directions.

b. There were two types of errors associated with the E$_t$<>ER$_n$ category. One type was predicted by the E$_t$<>ER$_n$ production in Figure 11 and involved reasoning that a change in a voltage drop always implies that the total voltage has changed. The other involved reasoning that, if total voltage remained the same, then all the voltage drops remained the same in spite of a change in one of the resistance values. This error was not predicted and, as shown in Table 8, required modification of the E$_t$<>ER$_n$ production to include the situation in which total voltage has remained the same.

c. R_n<>R_t corresponds to the overgeneralized production that says that total resistance is increased by increasing the value of a resistor or adding a resistor and is decreased by decreasing the value of a resistor or removing a resistor. This production leads to incorrect predictions about the effects of adding or removing resistors in a simple parallel circuit: Adding a resistor in parallel decreases total resistance; removing a parallel resistor increases total resistance.

d. Balancing errors occurred primarily on the combination circuit problem in which the resistance changes in one of the branches. The error seemed to result from having acquired a rule that a change in a part sometimes results in the opposite change in another part but at the same time having failed to understand the constraints required to apply this rule.

In summary, subjects' performance on qualitative problems was consistent with the predictions of the theoretical analysis. Subjects' failure to understand the circuit constraints was evidenced by errors that corresponded to the overgeneralized productions for reasoning about circuit relations (Figure 11). These rules omit explicit tests for conditions that must remain the same for a change to have the predicted effects and therefore lead to predictable errors when these conditions are not met.

Further, it was not the case that subjects simply did not know how to determine which variables remain constant for a given change. They knew about the inherent properties of batteries and resistors but lacked the procedures for incorporating this knowledge and making use of it. In other words, they knew that changing the applied voltage has no effect on resistance values and that changing a resistance value has no effect on the other resistance values or the applied voltage. However, they did not seem to understand why it was important to consider this information when reasoning about qualitative changes--especially since the rules they had were sufficient to answer most of the problems in the course correctly.

Conclusions

These results suggest that, even when students have the minimum components of knowledge required to solve problems in a domain available, this is not always sufficient. Unless the constraints associated with that knowledge are also made salient and incorporated into problem-solving procedures, students will (1) fail to make critical distinctions between the elements in a problem (e.g., the E=R and LG errors), (2) fail to recognize critical conditions for applying certain rules (e.g., the absence of "same" tests for resistance), and (3) overextend analogical models to reason incorrectly about relations between elements in the domain.

EXPERIMENT II

Purpose

The purpose of Experiment II was to determine whether the errors students made in Experiment I could be prevented by providing them with a model of a concrete analogy of circuit relations in which the constraints are salient and therefore more likely to be understood and used in solving problems. It was hypothesized that, if students are shown how problem solutions using the concrete analogy correspond to problem solutions in BE/E, they may be able to incorporate these constraints into their procedures for using

equations and rules. The model is an analogy of the steady-state constraints between current, voltage, and resistance; it is not intended to represent the processes by which circuits operate.

Model Description

Figure 14 shows a concrete analogy that constrains circuit relations and makes them salient. It involves simply a stack of chips that are divided equally among all the slots in boxes along a connected path. The number of chips in the stack corresponds to the total voltage. Each box corresponds to a resistor and the number of slots in a box indicates the resistance value. In Figure 14, the resistance values at R_1 and R_2 are 2 and 3 respectively. The values of the current and voltage drops are determined by dividing all the chips equally among the resistor slots--current corresponds to the number of chips in a single slot (in this case, 2) and the voltage drop across a resistor corresponds to the total number of chips in all its slots (the voltage drops across R_1 and R_2 and 4 and 6 respectively).

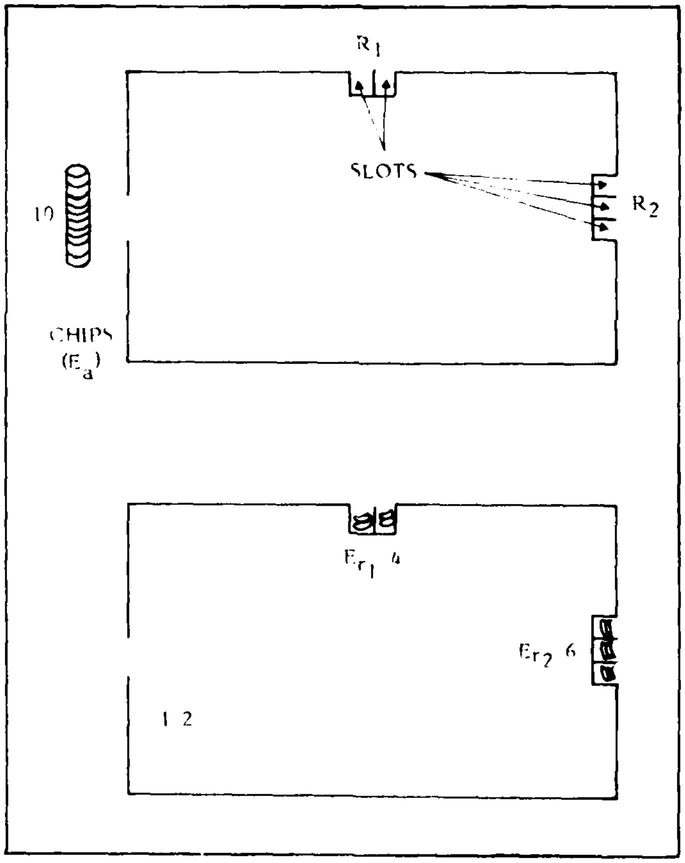

Figure 14. Model of analogy that constrains circuit relations and makes them salient.

Notice that the intrinsic properties of the objects in the chips analogy--the stack of chips and the boxes with slots--correspond to the intrinsic properties of the objects in an electrical circuit--the battery and the resistors. That is, changing the number of chips in the stack has no effect on the number of slots in the same way that changing the battery

has no effect on the resistance values. Similarly, changing the number of slots has no effect on the number of chips in the same way that changing a resistor has no effect on the battery. More important, however, is the fact that the procedure "divide the chips equally among the slots" explicitly considers these properties, thereby ensuring that all circuit constraints are satisfied:

1. The sum of the voltage drops is equal to the total voltage.

2. The amount of voltage dropped across a resistor is proportional to the amount of resistance.

3. Current is the same through each of the resistors.

Therefore, as described below, using the chips analogy to solve quantitative and qualitative problems should block the errors identified in the first experiment. Further, if, for some reason, a constraint were violated, this should be visible to the student and therefore more likely to be corrected.

In Experiment I, the two most common errors students made on quantitative problems were due to their failure to distinguish between voltage drops and resistance (E=R) and between local and global variables (LG). Since voltage drops and resistance are visibly distinct in the chips analogy, they should be less likely to be confused. For the same reason, subjects should be less likely to confuse local and global variables by, for example, determining the current at R_1 by dividing the total number of chips into the boxes at R_1. A case where all the chips had not been equally distributed among all the slots in the circuit should be immediately obvious.

The chips analogy should also prevent errors on qualitative problems that result from dropping the "same" condition from rules linking changes and effects. As mentioned above, the intrinsic properties of the objects in the chips analogy correspond to the intrinsic properties of batteries and resistors--increasing the stack of chips has no effect on the number of slots and increasing the number of slots has no effect on the total number of chips. Further, since these properties have a direct effect on the procedure "divide all the chips equally among the slots," they should be more likely to be noticed. Thus, if subjects are told, for example, that the resistance value of R_2 in Figure 14 has increased, they should be able to envision redistributing the chips and determine that increasing the number of slots in R_2 (1) decreases the number of chips in all the slots in the circuit, (2) decreases the number of chips for box R_3, (3) increases the number of chips for box R_2, and (4) has no effect on either the total number of chips or the number of slots in box R_1. This corresponds to the correct predictions that an increase in R_2 results in a decrease in current, a decrease in E_{R_1}, and an increase in E_{R_2}, but the total voltage and the resistance value of R_1 remain the same. Thus, the procedure for solving quantitative and qualitative problems using the chips analogy ensures that all circuit constraints are satisfied.

An additional rule is required to constrain the relations in parallel circuits, as shown by Figure 15, which shows the chips version of a simple parallel circuit. Recall that, in a parallel circuit, the full amount of applied voltage is dropped across each branch.

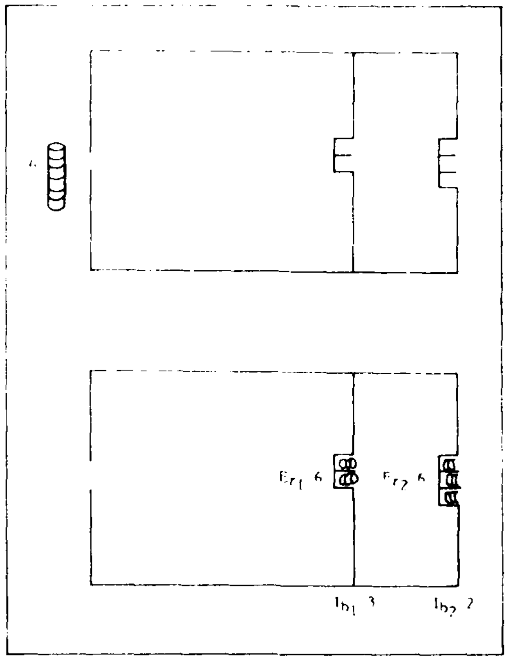

Figure 15. Chips version of a simple parallel circuit.

Therefore, the rule to distribute the stack of chips equally among all the slots must be modified to apply within each branch instead of across the entire circuit; that is, the entire stack of chips must be distributed separately within each branch. Thus, in Figure 15, the stack of six chips would be distributed once in branch 1 and again in branch 2. With this modification, all the relations between current, voltage, and resistance in parallel circuits are constrained. Further, if subjects are told that total current is the sum of the currents through each branch, they should be able to see why adding/removing a resistor in parallel increases/decreases current.

Method

Subjects

Subjects were five undergraduate males selected because their ASVAB test scores showed that their academic and career histories were similar to those of BE/E circuit population. None had studied basic electricity or was familiar with electric circuits.

Materials

The standard BE/E modules used in the last study were modified to include instruction on solving series and parallel circuit problems using the chips analogy. At the beginning of the modules on series circuits, subjects were shown a diagram like the one in Figure 14 and told that the stack of chips represented the total voltage, the boxes represented resistors, and the number of slots in a box indicated its resistance value. Then they were told that, to determine the current at any resistor, they would simply distribute the chips of total voltage equally among all the resistor slots in the circuit; the number of chips in any slot determines the current. The voltage drop across a resistor is determined by the total number of chips in all its slots.

Before students began the module on parallel circuits, they were shown a circuit like the one in Figure 15. They were told that:

1. In this case, the full amount of applied voltage is dropped across each parallel branch; therefore, the stack of chips is distributed separately within each branch.

2. The total current for a parallel circuit is the sum of the currents in the individual branches.

3. To find the equivalent resistance, they should simply find the total current and then divide the total voltage by the total current. The result would be the value of the equivalent resistance.

This procedure is algebraically equivalent to the reciprocal method formula for finding equivalent resistance given in the BE/E modules: $Req = 1 (1/R_1 = 1/R_2 + ...1/R_n)$.

After these introductions to the chips analogy, subjects were shown the mappings between the model and the corresponding circuit equations. Subjects then solved problems identical to the ones in the standard modules except that the standard circuit schematics were replaced by circuits with slots and chips.

The instruction on combination circuits introduced students to the standard circuit schematic; no further mention was made of the chips. Subjects were then asked to solve the same combination problems used in Experiment I. The only difference was that the multiple-choice format was replaced by an open-ended response format to increase the diagnosticity of the problems. If anything, this change should have made the problems more difficult.

Since the purpose of this second experiment was to determine whether subjects would make fewer errors using the chips analogy than the standard BE/E procedure, the instructional materials were shortened by eliminating problems on conversion of units, power, opens and shorts, and hands-on experience.

Procedure

The procedure was identical to that described for Experiment I. Subjects were studied individually approximately twice a week during protocol sessions that lasted anywhere from 1 to 2 hours. During each session, the subject paced himself through the contents of a BE/E module in the presence of an experimenter trained in taking protocol observations. Subjects had paper and pencil available at all times but were not permitted to use a calculator, in accordance with the Navy course regulations. Each session was tape-recorded and the subject was encouraged to think aloud while reading through the module and solving the problems. If a subject neglected to comment on how he had solved a particular problem, the experimenter prompted the subject with specific questions and/or described in detail the sequence of equations the subject had written down during problem solution. If a subject failed to respond to a problem, or responded incorrectly, the experimenter showed the subject the correct solution, as given in BE/E.

Analyses

Transcriptions of the verbal protocol tapes were analyzed to determine (1) the proportion of correct responses on quantitative and qualitative problems, and (2) the actual types of incorrect responses made by the subjects. These results were then used to

test the predictions that (1) the chips analogy would facilitate performance on series and parallel problems and (2) experience using the chips analogy to solve these problems would facilitate performance on combination problems.

Results

Quantitative Problems

The percentage of correct responses to quantitative problems, as shown in Table 9, indicates that the chips analogy did facilitate performance on series and parallel problems, as compared with the proportions correctly obtained in the first experiment (Table 5). However, errors persisted on problems that required solving for the voltage drop across a series resistor. The most frequent error was the LG error identified in the last study--using a global circuit value to solve for a local value when applying ohm's law. None of the errors involved confusions of voltage drops and resistance (E=R).

Table 9

Percentages of Correct Responses by Problem Type and Difficulty Level (Chips—Quantitative Problems)

Type of Circuit Problem	Number of Solution Levels					
	1	2	3	4	5	6
Series:						
I	100(6)	100(6)	--	--	--	--
E	100(5)	100(2)	--	--	--	--
R	100(3)	--	--	--	--	--
E_{R_n}	100(2)	100(1)	60(2)	80(1)	--	--
Parallel:						
I	100(4)	100(3)	--	--	--	--
E	--	--	--	--	--	--
R(1/...)	100(9)	--	--	--	--	--
E_{R_n}	100(3)	--	--	--	--	--
Combination:						
E_{Req}	--	--	--	--	--	100(1)
R_3	--	--	--	--	--	--
R_t	100(1)	90(2)	--	--	--	--
I_t	100(3)	--	--	--	--	--
E_{R_2}	100(1)	--	80(1)	80(1)	--	--
I_{R_3}	90(2)	60(1)	--	--	--	--

Note. The number in parentheses beside each percentage indicates the number of problems subjects solved of each type.

Since it was predicted that the LG error would not occur when solving problems using the chips analogy, protocols were examined closely to identify the source of difficulty. In fact, it appears that errors did not occur when students used the chips analogy; but, rather when they failed to apply the analogy. Two subjects in particular consistently preferred to focus only on the equations when solving problems. When asked to solve the problem again using the chips analogy, these subjects always performed correctly. Although the other three subjects made few explicit references to the chips analogy, they solved most of the problems correctly and did not make the LG error. One interpretation of these results is that experience solving problems using the chips analogy enabled subjects to incorporate the relevant constraints into their procedures for solving problems using equations in ways that no longer required explicit application of the chips analogy.

Qualitative Problems

Subjects' performance on qualitative problems was generally consistent with the findings just described. As shown in Table 10, the percentages of correct responses on series and parallel problems were quite high. The only errors occurred on problems that concerned the effects of a change in a series resistor and involved reasoning that all the voltage drops would increase if current increases, or all the voltage drops would decrease if current decreases. In two cases, subjects said they had not used the chips analogy. However, in the other two cases, subjects did apply the chips analogy but seemed to have difficulty envisioning the qualitative change in the number of chips in the resistor box that had changed. When told they were wrong, they had to use paper and pencil to redistribute the chips. However, once they did this, they seemed to understand why the voltage drop had to change in the direction of the increase in resistance and solved subsequent problems of this type correctly. Thus, the chips analogy seemed to facilitate subjects' ability to reason about the effects of qualitative changes in series and parallel circuits, as well as their performance on the combination problems. The main exception to this claim occurred primarily on problems that required students to determine how a change in one of the branch resistances affects the current within that branch. Even subjects who had used the chips analogy to solve earlier problems failed to realize that if, for example, current increased in one of the two parallel branches but total current decreased, then the current had to decrease in the other branch. Subjects were able to reason correctly about the increase in current in the branch in which resistance has stayed the same but did not immediately see how to constrain the change in current in the other branch. One possible reason they failed to consider total current to constrain the branch current may have been that this is only necessary on this particular problem and is not required on any series or parallel problem they solved using the chips analogy.

Conclusions

The results of Experiment II suggest that the chips analogy facilitated performance on series and parallel problems by making circuit constraints more salient and by providing subjects with simple procedures that take those constraints explicitly into account. Further, it appears that showing subjects how this analogy corresponds to circuit equations and qualitative rules may have enabled them in some cases to incorporate these constraints into procedures for solving problems correctly without explicitly using the analogy. However, results also suggest that further research is needed to identify ways of connecting students' understanding of the constraints on procedures for manipulating concrete materials to their understanding of analogous procedures for manipulating symbols.

Table 10

Percentages of Correct Responses by Problem Type and Difficulty Level
(Chips--Qualitative Problems)

Type of Circuit Problem	Predicted Effect					
	I_t	I_{R_n}	E_t	E_{R_n}	R_t	R_n
<u>Series</u>:						
E_t	1.00(8)	--	--	1.00(3)	1.00(3)	--
R_t	1.00(7)	--	1.00(1)	--	--	--
R_n	1.00(6)	--	1.00(1)	.87(6)	1.00(1)	--
<u>Parallel</u>:						
E_t	1.00(5)	1.00(1)	--	1.00(2)	1.00(3)	--
R_n	1.00(13)	1.00(2)	1.00(2)	1.00(3)	1.00(7)	1.00(2)
<u>Combination</u>:						
E_t	1.00(6)	1.00(6)	--	1.00(5)	1.00(6)	--
R_1	1.00(4)	.80(4)	1.00(4)	.90(4)	1.00(4)	--
$R_{2,3,4}$	1.00(6)	.70(6)	1.00(6)	.88(6)	1.00(6)	--

<u>Note</u>. The number in parentheses beside each percentage indicates the number of problems subjects solved of each type.

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